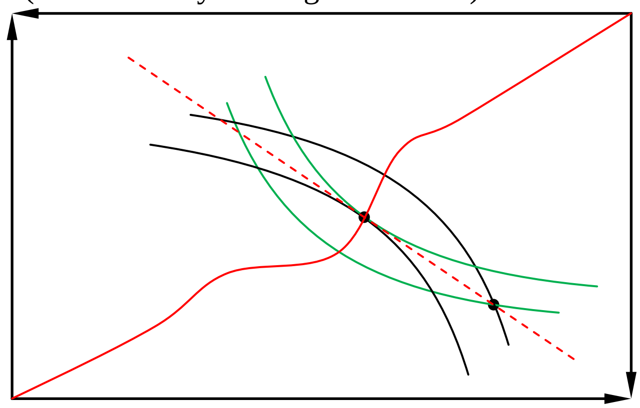
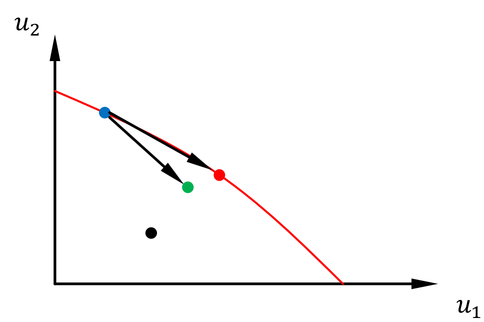
Introduction

- First Welfare Theorem (illustration by the Edgeworth Box)



  - The competitive equilibrium (the tangency) is Pareto efficient unless…
    - Public goods (positive externality)
    - Externality (negative ones, e.g. pollution)
      - Negative externalities are related to not well-defined property rights
    - Unsecure property rights
    - Non-competitive behavior
    - Informational issues (e.g. moral hazard and adverse selection)
  - All of the above black conditions can be remedied by governmental intervention / regulation. However, for the green condition, the government is not necessarily in an advantageous position to intervene.

  - The Utility Possibility Frontier (UPF)



    - The UPF is essentially the contract curve.
    - Government spending may be used to improve efficiency in the market, OR, it can also be used as redistribution (towards egalitarian ends or as political transfers).

Review of Canadian Tax System

- Size of government (i.e. spending / GDP) is about 30%
  - 60% - 70% of government expenditure is on redistribution (not on *public goods*). And this is true for all levels of government.

- Revenue for the Canadian Government: close to 50% is from income taxes (personal and corporate)

2.1 Tax Avoidance and Excess Burden v.s. Neutral Taxation

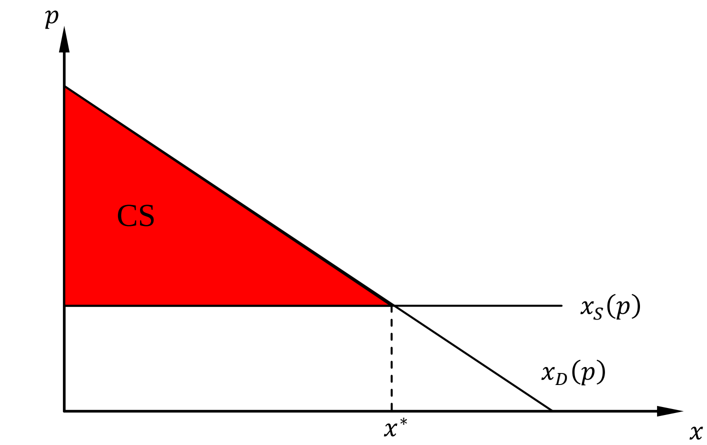
- Lump sum tax is the tax that cannot be avoided, either legally or illegally
  - This is the tax system assume in a course on public expenditure

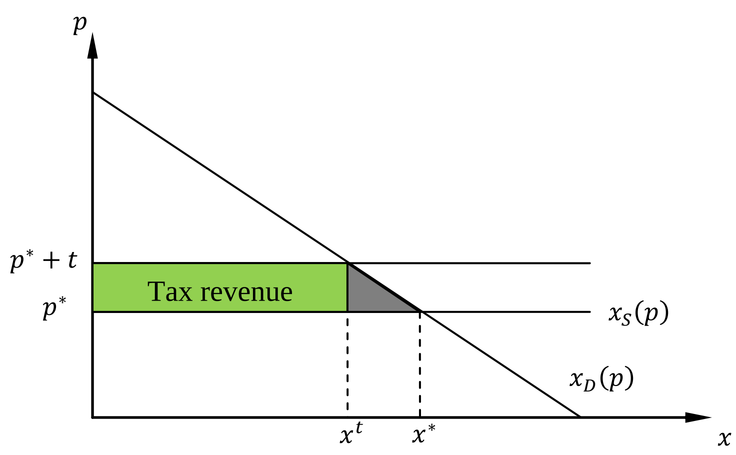
- Example: the window tax
  - Suppose a city has 100 people, 10 rich and 90 poor, and need to finance $10000 of spending.
    - Lump sum tax: 100 per person $\rightarrow$ 10000
      - This is assuming we cannot identify whether a person is rich or poor
    - Suppose the rich people each has one window, while the poor has no window. Then a lump sum tax would be 1000 per window.
      - In the long run, 5 individuals block their window $\rightarrow$ this is avoidance activity. The lump sum tax would increase to 2000 per window.
      - Even though 10000 tax revenue is still generated. But there is welfare loss in the form of people's avoidance activity; that is, those who blocked their windows don't get to enjoy utility from having a window.
  - Readings:
    - Must-read: Diamond and McFadden (1974); Auerbach (1985)
    - For PhD: Bergstrom, Blume, and Varian (1986);

- "Neutral" taxation may refer to different things in different settings. Sometimes "neutral" basically means that the tax is non-distortionary of a certain aspects of a decision.

2.2 Consumption Taxes

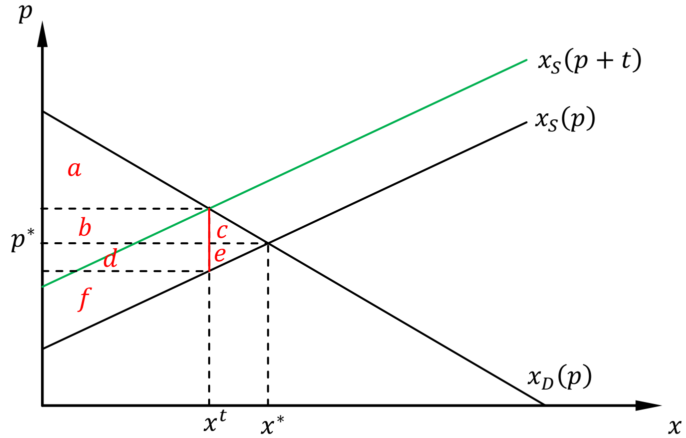
- Suppose there is 1 good, and 1 tax. Demand for good x is given by $x_D(p)$ with $x_D'(p) < 0$. Price p is competitive, $x_S'(p) \geq 0$.





- Taxes and elasticity
  - If demand is elastic, avoidance activity caused by the increase in tax will be high. Therefore, DWL will be high, and tax revenue generated will be small.
  - If demand is inelastic, avoidance activity resulting from the increase in tax is small. Thus, DWL will be small, and tax revenue generated will be high.

- Tax incidence – general equilibrium analysis of taxes
  - Atkinson and Stiglitz, lecture 6.1-6.2



- No tax:
  - $CS = a + b + c$
  - $PS = d + e + f$
- With tax:
  - $Tax\ Rev = b + d$
  - $DWL = c + e$
    - % of tax paid by consumer: $\frac{b}{b+d}$
    - The more elastic the demand, the smaller the share of the tax burden is put on consumers.

- 2 goods
  - Consumer surplus cannot be used here as a measure of welfare (unless the utility function is homothetic, or if the two goods are neither complements or substitutes)
  - Measuring excess burden in several markets:
    - $N + 1$ Goods: $\{x_0; x_1, \dots, x_n\}$ where
      - x_0 is non-taxable (consider it as leisure or home production) with $p_0 = 1$
      - x_i for $i > 0$ are market goods, taxable, with $q_i = p_i(1 + t_i)$ being after-tax price, and p_i is constant (i.e. free entry with constant marginal cost)
      - Price vector: $Q = [1 \ \ q_1 \ \ \cdots \ \ q_n]$
      - Income for an individual is M
      - Indirect utility function: $v(Q, M)$
      - Budget constraint: $M = x_0 + \sum_{i=1}^{n} p_i(1 + t_i)\, x_i$
        - Without x_0, the optimal way is to set a uniform tax on all goods. This is basically a world where lump sum tax is available. However, the non-taxable x_0 prevents the lump sum tax from being used. Therefore, in this world with non-taxable goods, all taxes are going to be distortionary.

Consumption Taxes (cont'd)

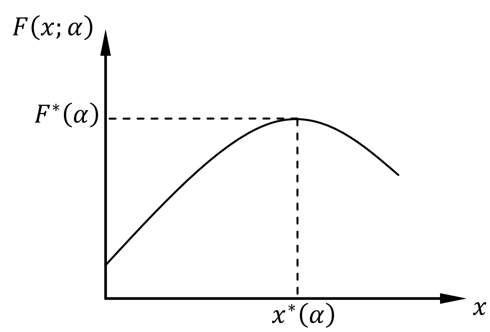
- Recall the environment:
  - Goods $X = \{x_0; x_1, \dots, x_n\}$;
  - after tax prices $Q = \{1; q_1, \dots, q_n\}$ where $q_i = p_i + t_i$;
  - Income m
  - Utility is $u(x_0, x_1, \dots, x_n)$
  - Indirect utility is $v(Q, m)$
  - Budget constraint: $m = x_0 + \sum_{i=1}^{n} q_i x_i$

- Note on indirect utility function

$$\max_x F(x; \alpha), \qquad with\ FOC \quad F_x(x; \alpha) = 0$$

Let the solution be $x^*(\alpha)$.

$$F^*(\alpha) = F(x^*(\alpha); \alpha), \qquad \frac{\partial F^*(\alpha)}{\partial \alpha} = F_\alpha(x^*(\alpha); \alpha) + \underbrace{F_x(x^*(\alpha); \alpha)}_{=0} \frac{\partial x^*(\alpha)}{\partial \alpha}$$



- Agent's problem

$$\max_{x_0, x_1, \dots, x_n} u(x_0, x_1, \dots, x_n), \qquad s.t.\ \ x_0 + \sum_{i=1}^{n} q_i x_i \le m$$

FOC:

$$u_{x_0} = \lambda$$
$$u_{x_i} = \lambda q_i$$
$$x_0 + \sum_{i=1}^{n} q_i x_i = m$$

Then, $x_0(Q, m)$ and $x_i(Q, m)$for $i \in \{1, \dots, n\}$. Thus the indirect utility is

$$v(Q, m) = u\big(x_0(Q, m), x_1(Q, m), \dots, x_n(Q, m)\big)$$

Take total derivative of $v(\cdot)$ (with respect to the price of good j):

$$dv = \sum_{i=0}^{n} u_{x_i}(\cdot) \frac{\partial x_i(\cdot)}{\partial q_j} dq_j + \sum_{i=0}^{n} u_{x_i}(\cdot) \frac{\partial x_i(\cdot)}{\partial m} dm$$

Use the FOC, where $u_{x_i} = \lambda q_i$:

$$\frac{dv}{\lambda} = \underbrace{\sum_{i=1}^{n} q_i \frac{\partial x_i(\cdot)}{\partial q_j} dq_j + \frac{\partial x_0(\cdot)}{\partial q_j} dq_j}_{-x_j(Q,m)dq_j} + \underbrace{\sum_{i=1}^{n} q_i \frac{\partial x_i(\cdot)}{\partial m} dm + \frac{\partial x_0(\cdot)}{\partial m} dm}_{\partial m}$$

Recall that the budget constraint is

$$x_0(Q,m) + \sum_{i=1}^{n} q_i x_i(Q,m) = m$$

Differentiate w.r.t q_j

$$\frac{\partial x_0(Q,m)}{\partial q_j} + \sum_{i=1}^{n} q_i \frac{\partial x_i(Q,m)}{\partial q_j} + x_j(Q,m) = 0$$

The ***money metric of utility***:

$$\frac{\partial v}{\lambda} = \frac{\text{change in utility}}{\text{shadow price}} = \frac{\Delta\text{utility}}{\text{utility/dollar}}$$

- Note however that the above analysis only works when the changes are <u>marginal</u>. If changes are not marginal, the envelope theorem does not work.
- This leads to the consideration of expenditures

❖ The Expenditure Minimization Problem

$$\min x_0 + \sum_{i=1}^{n} q_i x_i\,, \qquad s.t.\;\; u(x_0; x_1, \dots, x_n) \geq \bar{u}$$

The solutions, $x_0(Q,\bar{u})$ and $x_i(Q,\bar{u})$, are the ***compensated demand***.

- The ***Slutsky equation***:

$$\frac{\partial x_i}{\partial q_i} = \left.\frac{\partial x_i}{\partial q_i}\right|_{\bar{u}} - x_i \frac{\partial x_i}{\partial m}$$

- The ***expenditure function***:

$$e(Q,\bar{u}) = x_0(Q,\bar{u}) + \sum_{i=1}^{n} q_i x_i(Q,\bar{u})$$

- ***Compensated Variation*** (***CV***) equivalent compensation:

$$CV = e(Q^1,\bar{u}^0) - e(Q^0,\bar{u}^0)$$

This is how much I need to give you, so that you are equally well off at the new price Q^1 compared to the old price Q^0

- ***Equivalent Variation*** (***EV***)

$$EV = e(Q^1,\bar{u}^1) - e(Q^0,\bar{u}^1)$$

This is how much you want to "bribe" the government in order to avoid the change in prices (e.g. due to changes in taxes).

3.1 Optimal Taxation of Goods

- Value added tax

	Value (Price)	Value Added	VAT (10%)	Sales (10%)
A	10	10	1	0
B	20	10	1	0
C	50	30	3	0
Final Good	100	50	5	10
	100	100	10	10

  - Superiority of VAT
    - It does not distort the behavior of buying input v.s. buying final good. In other words, the VAT does not change the price ratio between intermediate and final goods
      - In contrast, a sales tax may provide incentive to engaging in the production of intermediate goods (e.g. instead of going to a restaurant, people may start cooking for themselves, even though the dislike cooking)
    - VAT also limit the incentive for tax evasion
      - With sales tax, since taxes are only paid at the end sales, there is a huge incentive for tax evasion → there is a $10 to be split between consumers and firm D.
      - With VAT, since the firms paid taxes in purchasing the inputs. Take firm D as an example. It has paid 3 dollars tax in purchasing input from firm C. So the surplus to be split between firm D and consumer (if they engage in tax evasion) is only $5.

- Ramsey Rule
  - Environment: 1 consumer, n goods
  - Suppose there are no non-taxable goods
  - p_i is producer i's price
  - t_i is tax on good i
  - $q_i = p_i + t_i$

  - Agent's objective

$$\max_{(x_1,\dots,x_n)} u(x_1,\dots,x_n)\,,\qquad s.t.\ \sum_{i=1}^{n} q_i x_i = w$$

FOCs: $u_{x_i}(\cdot) = \lambda q_i \ \rightarrow$ solve $x_i(Q,w)$. This leads to an indirect utility function

$$v(Q,w) = u\big(x_1(Q,w),\dots,x_n(Q,w)\big)$$

  - Problem of the government

$$\max_{t_1,\dots,t_n} v(Q,w)\,,\qquad s.t.\ \sum_{i=1}^{n} t_i x_i(Q,w) \geq G$$

    - Note: $\frac{\partial x_i(Q,w)}{\partial t_k} = \frac{\partial x_i(Q,w)}{\partial q_k}$

Government's Lagrangian

$$\mathcal{L} = u\big(x_1(Q,w),\dots,x_n(Q,w)\big) + \psi[t_1 x_1(Q,w) + \cdots + t_n x_n(Q,w) - G]$$

- Effect of tax on good i:

$$u_{x_i}(\cdot)\frac{\partial x_i(\cdot)}{\partial t_i}+\sum_{j\neq i}u_{x_j}(\cdot)\frac{\partial x_j(\cdot)}{\partial t_i}+\psi\left[x_i(Q,w)+t_i\frac{\partial x_i(\cdot)}{\partial t_i}+\sum_{j\neq i}t_j\frac{\partial x_j(\cdot)}{\partial t_i}\right]=0$$

Using FOC from agent's problem $u_{x_i}(\cdot)=\lambda q_i$:

$$-\sum_{j\neq i}\lambda q_j\frac{\partial x_j(\cdot)}{\partial t_i}=\psi\left[x_i(Q,w)+\sum_{j=1}^{n}t_j\frac{\partial x_j(\cdot)}{\partial t_i}\right]$$

But $\frac{\partial x_j(\cdot)}{\partial t_i}=\frac{\partial x_i(Q,w)}{\partial q_k}$:

$$-\lambda-\sum_{j\neq i}q_j\frac{\partial x_j(\cdot)}{\partial q_i}=\psi\left[x_i(Q,w)+\sum_{j=1}^{n}t_j\frac{\partial x_j(\cdot)}{\partial q_i}\right]$$

From agent's BC:

$$\sum_{j=1}^{n}q_jx_j(\cdot)=w$$

So

$$x_i(\cdot)+\sum_{j=1}^{n}q_j\frac{\partial x_j(\cdot)}{\partial q_i}=0$$

$$\lambda x_i(\cdot)=\psi\left[x_i(\cdot)+\sum_{j=1}^{n}t_j\frac{\partial x_j(\cdot)}{\partial q_i}\right]\Leftrightarrow\ \left[\frac{\lambda}{\psi}-1\right]x_i(\cdot)=\sum_{j=1}^{n}t_j\frac{\partial x_j(\cdot)}{\partial q_i}$$

$$\Leftrightarrow\ \left[\frac{\lambda}{\psi}-1\right]=\sum_{j=1}^{n}\frac{t_j}{q_i}\cdot\frac{q_i}{x_i(\cdot)}\cdot\frac{\partial x_j(\cdot)}{\partial q_i}$$

Suppose utility is separable in the n goods: $u(x_1,\dots,x_n)=u_1(x_1)+\cdots+u_n(x_n)$, and $\frac{\partial x_j(\cdot)}{\partial q_i}=0$ for all $j\neq i$. Then,

$$\left[\frac{\lambda}{\psi}-1\right]=\frac{t_i}{q_i}\epsilon_i,\qquad \epsilon_i=\frac{\partial x_i(\cdot)}{\partial q_i}\cdot\frac{q_i}{x_i(\cdot)}$$

For any $i\neq j$,

$$\epsilon_i\frac{t_i}{q_i}=\epsilon_j\frac{t_j}{q_j}\ \Rightarrow\ \frac{t_i/q_i}{t_j/q_j}=\frac{\epsilon_i^{-1}}{\epsilon_j^{-1}}$$

Ramsey Rule (cont'd)

- Recap from last class:
  - No non-taxable good
  - Fully separable utility function: $\partial x_i / \partial q_j = 0$ for all $i \neq j$
  - Then, optimal consumption taxes are

$$\boxed{\frac{\tau_i}{\tau_j} = \frac{t_i/q_i}{t_j/q_j} = \frac{1/\epsilon_i}{1/\epsilon_j}}, \qquad \epsilon_i = \frac{q_i}{x_i} \cdot \frac{\partial x_i(\cdot)}{\partial q_i}$$

    - Here τ_i is the <u>percentage</u> tax on good i, and t_i is the <u>amount</u> of tax of good i

- Suppose there is a non-taxable good

$$u(x, \ell) = u_1(x_1, \ell) + \cdots + u_n(x_n, \ell)$$
$$\frac{\partial x_i(\cdot)}{\partial q_j} = 0, \qquad \forall i > 1, \;\; i \neq j$$
$$\frac{\partial \ell(\cdot)}{\partial q_j} \gtrless 0$$

$$\left[\frac{\psi - \lambda}{\psi}\right] = \tau_i \left[\epsilon_{q_i}^i - \epsilon_{q_i}^0\right], \qquad \begin{cases} \epsilon_{q_i}^0 = -\dfrac{\partial \ell}{\partial q_i} \cdot \dfrac{q_i}{\ell} \\ \epsilon_{q_i}^i = -\dfrac{\partial x^i}{\partial q_i} \cdot \dfrac{q_i}{x_i} \end{cases}$$

  - Note that

$$\epsilon_{q_i}^0 > 0 \;\; \Rightarrow \;\; \ell \text{ is a complement to } x_i$$
$$\epsilon_{q_i}^0 < 0 \;\; \Rightarrow \;\; \ell \text{ is a substitute to } x_i$$

- Corlett-Hague Rule

$$\boxed{\frac{\tau_i}{\tau_j} = \frac{\epsilon_{q_j}^j - \epsilon_{q_j}^0}{\epsilon_{q_i}^i - \epsilon_{q_i}^0}}$$

	$\epsilon_{q_i}^0 > 0$	$\epsilon_{q_i}^0 < 0$
$\epsilon_{q_i}^i$ high	?	Very low tax rate
$\epsilon_{q_i}^i$ low	Very high tax rate	?

Income Taxation

- Budget constraint: $w = x_0 + q_1 x_1 + \cdots + q_n x_n$
  - Income tax is like a uniform taxation that taxes all commodities at the same rate!!!
  - Uniform taxes distorts no relative prices between goods except non-taxable

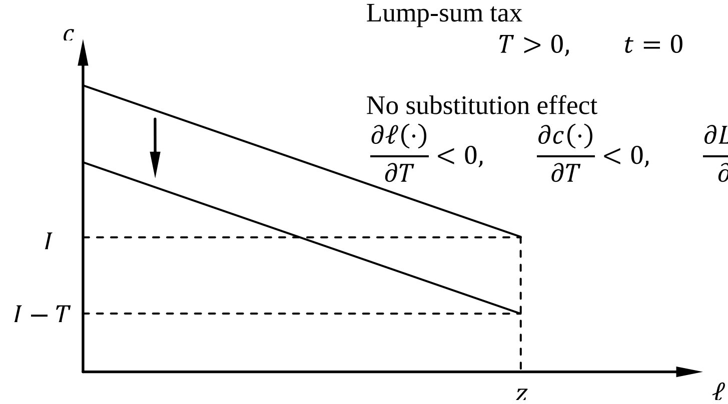
- Income taxes v.s. consumption taxes
  - It is easier to make income taxes progressive than to increase progressivity in consumption taxes
  - It is however easier to evade income taxes (collusion between employer and employee)

- Taxation and Labor Supply
  - C.f. Boadway and Kitchen (1999) "Canadian Tax Policy" *Canadian Tax Paper* No. 103, Chapters 1 and 2

- Simple labor supply model
  - 1 agent (no interpersonal comparison)
  - Preference: $u(c,\ell)$, c for consumption and ℓ for leisure
    $$u_c > 0, \qquad u_{cc} < 0, \qquad u_\ell > 0, \qquad u_{\ell\ell} < 0$$
  - Price of consumption equal 1
  - Total time $z = L + \ell$, where L is labor supply
  - Outside income: I
  - Lump-sum tax: T
  - Wage tax: t
  - Budget constraint:
    $$c = (1-t)w\underbrace{[z-\ell]}_{L} + I - T \quad \Leftrightarrow \quad (1-t)w\ell + c = (1-t)wz + I - T$$
  - Problem of the agent:
    $$\max_{c,\ell} u(c,\ell), \qquad s.t.\ (1-t)w\ell + c = (1-t)wz + I - T$$

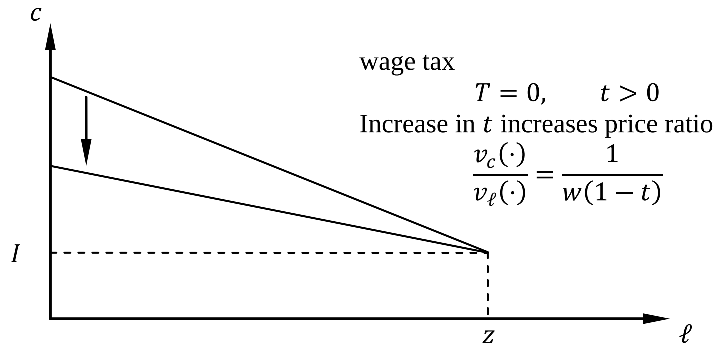
    FOC:
    $$\left.\begin{aligned} u_c(\cdot) &= \lambda \\ u_\ell(\cdot) &= (1-t)w\lambda \end{aligned}\right\} \Rightarrow \frac{u_c}{u_\ell} = \frac{1}{(1-t)w}$$

    From there we can solve for $\ell(t,T)$, $c(c,T)$, and $L(t,T) = z - \ell(t,T)$. Thus, we get the indirect utility $v(t,T)$.



Lump-sum tax
$$T > 0, \qquad t = 0$$

No substitution effect
$$\frac{\partial \ell(\cdot)}{\partial T} < 0, \qquad \frac{\partial c(\cdot)}{\partial T} < 0, \qquad \frac{\partial L(\cdot)}{\partial T} > 0$$

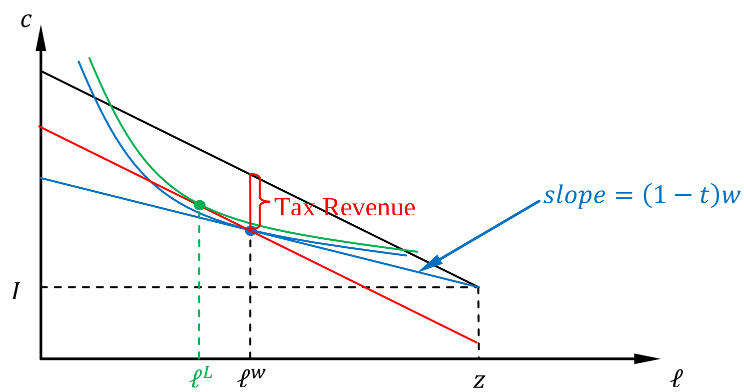


- Substitution effect
$$\left.\frac{\partial \ell(t,T)}{\partial t}\right|_{\bar{u}} > 0, \qquad \left.\frac{\partial c(t,T)}{\partial t}\right|_{\bar{u}} < 0$$
- Income effect
$$\frac{\partial \ell(t,T)}{\partial\ income} < 0, \qquad \frac{\partial c(t,T)}{\partial\ income} < 0$$
- Total effect
$$\frac{\partial \ell(t,T)}{\partial t} \overset{?}{\gtrless} 0, \qquad \frac{\partial c(t,T)}{\partial t} < 0$$

➢ Slutsky equation and labor supply
$$\frac{\partial L(t)}{\partial w} = \underbrace{\left.\frac{\partial L(t)}{\partial\ income}\right|_{\bar{u}}}_{\oplus} + \underbrace{L(t)\frac{\partial L(t)}{\partial\ income}}_{\ominus}$$

$$\frac{\partial L(t)}{\partial t} = \underbrace{\underbrace{\frac{\partial L(t)}{\partial W}}_{\oplus}\underbrace{\frac{\partial W}{\partial t}}_{\ominus}}_{\ominus} + L(t)\underbrace{\underbrace{\frac{\partial L(t)}{\partial\ income}}_{\ominus}\underbrace{\frac{\partial\ income}{\partial t}}_{\ominus}}_{\oplus}, \qquad W = (1-t)w$$

➢ Start with wage tax and replace it with a revenue equivalent lump-sum tax



➢ Excess burden is proportional to changes in labor supply
- If $\partial L(w)/\partial w$ is large, then distortion is large

- Problem: most studies show close to zero elasticity of L
  - Studies on hours worked (by males) and labor participation

- Need to look at other channel after tax wage influence labor decision
  - Secondary earner (female labor hours) $\rightarrow$ household decision
  - Occupational choice $\rightarrow$ wages are endogenous
  - Effort and promotion $\rightarrow$ again, endogenous wages
  - These suggest that we should look at total (household) income elasticity instead

Income Taxes (cont'd)

- Total income elasticity may not fully reflect the effect of income tax

- Taxes may have an impact on taxable income
  - Tax exempt compensations
    - E.g. health insurance,
  - Tax-advantaged compensations
    - E.g. pension, stock option
  - Deductible consumption
    - E.g. charitable contribution, political donation
  - Tax avoidance activities (aggressive tax planning)
    - E.g. paying high rates to accountants and lawyers just to find ways to (legally) avoid paying taxes

  - In all of these cases, wages and labor supply are not changed, but distortions do occur.
  - These suggest looking at changes of <u>total taxable income</u> (c.f. Feldstein 1995)

- How to measure taxable income elasticity
  - First, need data with some variation. Variation can be from…
    - Different "region" (e.g. municipalities, provinces, countries)
    - Change in policy
      - Better yet: policies do not change for all individuals (control & treatment groups)

- Feldstein (1995)
  - Tax Reform Act of 1986
    - Data panel for tax payers from 1985 to 1988
  - Model:

$$\ln \underbrace{Y_{ith}}_{\substack{\text{taxable}\\ \text{income}}} = \underbrace{\alpha_i}_{\substack{\text{income}\\ \text{level effect}}} + \underbrace{B_t}_{\substack{\text{aggregate}\\ \text{time effect}}} + \gamma \ln\left(1 - \underbrace{\tau_{it}}_{\text{tax rate}}\right) + \epsilon_{ith}$$

    where$t$ is time, i is income bracket, h is household
  - Since there's a policy change in 1986, can estimate the effect using a difference-in-difference approach

  - Problems with this approach
    - There is more than one tax base
      - Personal v.s. corporate taxes
    - Acceleration of deferral of declaring income (time shifting)
      - E.g. RRSP.

- Tax-free savings account
  - Imagine a case with two individuals, A and B, and the marginal tax rates are

$$\begin{array}{ll} 0\%, & 0 - 39999 \\ 10\%, & 39999 - 79999 \\ 25\%, & 79999 \text{ and above} \end{array}$$

	A		B	
Time	Income	Tax	Income	Tax
t	40000	4000	80000	20000
$t+1$	40000	4000	0	0
Total	80000	8000	80000	20000

Progressivity of a Tax System

- ***Marginal tax rate***: tax rate on the last dollar
  - May be useful for thinking about how individuals respond in terms of decisions, but not so good in judging the progressivity of a tax system

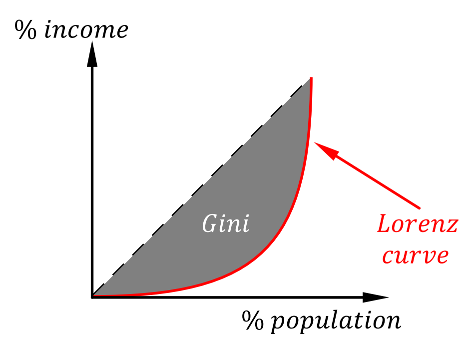
- ***Average tax rate***: total tax paid as a proportion of total income, $ATR = \frac{tax\ paid}{total\ income}$

- Lump-sum taxes:
  - $MTR = 0$
  - $ATR = \frac{T}{w(z-\ell)+I} = \frac{T}{M}$, with $\frac{\partial ATR}{\partial M} < 0$
    - The tax system is *regressive*, the higher the income, the smaller the average tax rate

- Wage taxes (assume $I = 0$):
  - $MTR = t$
  - $ATR = \frac{tw(z-\ell)}{w(z-\ell)} = t$ with $\frac{\partial ATR}{\partial M} = 0$
    - Such a tax system is *neutral*.
    - Note that if $I \neq 0$ and such outside income is non-taxable, then the system is regressive when the source of increase in income is from increase in I

- Two principles of equity
  - ***Horizontal equity***: individuals with identical pre-tax income should end up with identical after-tax income
  - ***Vertical equity***: individuals with higher pre-tax income should end up with still higher after-tax income

- Measure of inequality
  - ***Equivalence scales***. Allows comparison between individuals/households of differing circumstances, e.g. those with same income but different number of children, etc.
  - Measures of inequality.
    - ***Lorenz curve***. Denote $x \in X$ as the income support, $f(x)$ is the income distribution
      $$\frac{1}{F(\bar{x})}\int_0^{\bar{x}} xf(x)dx = \text{average income up to } \bar{x}$$
      Look at the conditional average income as a proportion of the unconditional income for all values of $\bar{x}$ from 0 to the highest income: $E(x|\bar{x})/E(x)$.



    - ***Gini coefficient***. Area between a perfect egalitarian society (45 degree line) and the Lorenz curve.

- Linear progressive tax
  - Tax rate $t\%$ on all income
  - Base exemption is e dollars
  - Budget constraint:

$$c = \begin{cases} w(z-\ell) & \text{if } c \leq \ell \\ (1-t)[w(z-\ell)-e]+e & \text{if } c > l \end{cases}, \quad \text{[exemption]}$$

$$= \begin{cases} w(z-\ell) & \text{if } c \leq \ell \\ (1-t)w(z-\ell)+te & \text{if } c > l \end{cases}, \quad \text{[tax credit]}$$

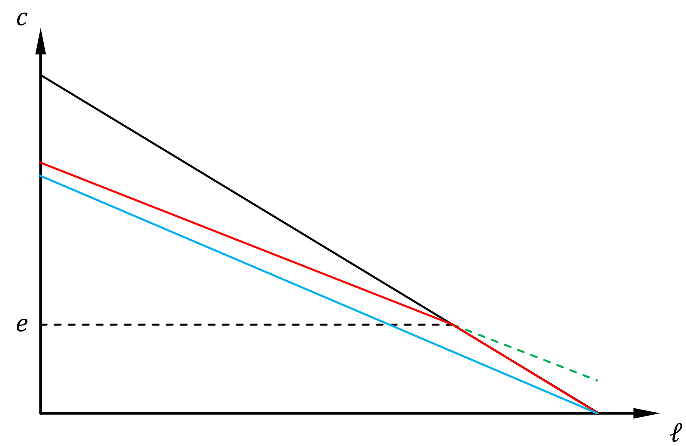
  - Average tax rate:

$$ATR = \begin{cases} 0 & \text{if } c \leq \ell \\ \dfrac{t[w(z-\ell)-e]}{w(z-\ell)} & \text{if } c > 0 \end{cases}$$

$$= \frac{t(M-e)}{M}$$

$$\frac{\partial ATR}{\partial M} = \frac{tM - tM + te}{M^2} = \frac{te}{M^2} > 0$$

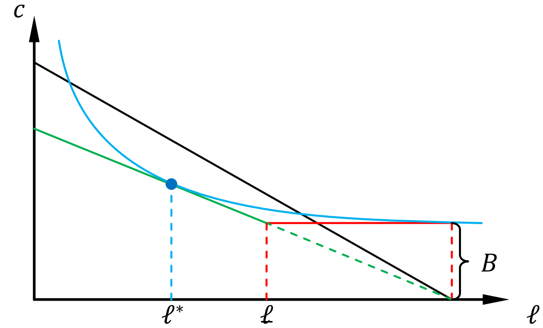
Now we have progressivity. The higher the exemption level is, the more progressive the tax system.



  - A trade-off between progressivity and distortion:
    - Progressivity is increased by increasing e,
    - But the higher the e, the higher the marginal tax rate on the high income earner, and hence the more distortion the tax system is creating.

- Redistribution
  - Non-targeted (universal): healthcare
  - Targeted (at certain individual characteristics, e.g. income level): HST refund

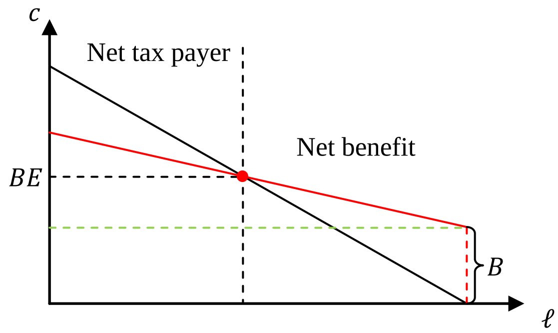
- Example: welfare system
  - Imagine there is a group of agents with $\ell = z$ (or $L = 0$, unable to work)
  - In a targeted system,
    - $B = \begin{cases} \text{benefit} & \text{if } L = 0 \\ 0 & \text{if } L > 0 \end{cases}$ and income is taxed at rate t
    - Budget constraint: $c = \begin{cases} B & \text{if } \ell = z \\ (1-t)w(z-\ell) & \text{if } \ell < z \end{cases}$



- If preference were described by the blue indifference curve, no one would work below the level ℓ^*
- A targeted system creates a discontinuity in the budget constraint
- Can think of $\ell > \ell^*$ as a kind of "poverty trap"

Welfare System (cont'd)

- Non-targeted system:
  - Everyone gets benefit B
  - Everyone pays tax rate t
  - $ATR = \frac{twL-B}{wL} = \frac{tM-B}{M}$



BE is the break even point:
c such that$tw(z-\ell) = B$

  - This system does not generate a discontinuity in the budget constraint, so there's no distortion in terms of inducing "poverty traps" by encouraging people near the cutoff not to work
  - But to balance budget, this system needs to tax income at a much higher rate than in the targeted system, because benefit B goes to everybody.
    - Due to a higher tax rate, substitution effect is higher (tax incidence in the high income earners is higher)
  - Every recipient is less distorted in the universal system than in the targeted system
  - The universal system creates too much distortion on the rich, by charging a higher marginal tax rate to them. But it reduces the distortion on the poor, by not inducing them to work zero hours.
  - Substitution effect are the same for both net payers and net recipients in a universal system: both types would substitute towards more leisure
  - Income effect for the net payers is negative, so there's pressure for them to work more hours (i.e. less leisure). So income and substitution effects are opposite for the rich. Income effect for net recipients is positive, and goes in the same direction as substitution effect.

Taxation and Saving

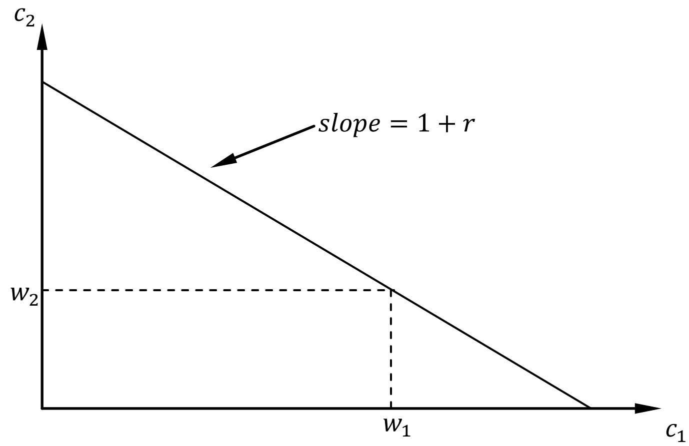
- One agent who live for N periods, where $N \in \{2, \dots, \infty\}$
- Utility: $u(c_1, \dots, c_N) = u(c_1) + \delta u(c_2) + \delta^2 u(c_3) + \cdots$, with $\delta \in [0,1]$
  - $MRS_{12} = \delta \frac{u'(c_2)}{u'(c_1)}$
- Income profile: $W = \{w_1, \dots, w_N\}$
- Interest rate: r

Taxation and Saving (cont'd)

- Budget constraint
  - Define s_t as saving at time t
  - At time t, budget constraint is

$$w_t + s_{t-1}(1+r) = c_t + s_t$$

$$\Rightarrow \underbrace{\sum_{t=0}^{n} \frac{w_t}{(1+r)^t}}_{\text{PV of income}} = \underbrace{\sum_{t=0}^{n} \frac{c_t}{(1+r)^t}}_{\text{PV of consumption}}$$



- Lump-sum taxes: $T_1, \ldots, T_n$
  - Budget constraint

$$\sum_{t=0}^{n} \frac{w_t - T_t}{(1+r)^t} = \sum_{t=0}^{n} \frac{c_t}{(1+r)^t}$$

$$\sum_{t=0}^{n} \frac{w}{(1+r)^t} - \underbrace{\sum_{t=0}^{n} \frac{T_t}{(1+r)^t}}_{G} = \sum_{t=0}^{n} \frac{c_t}{(1+r)^t}$$

  - Government budget constraint

$$\sum_{t=0}^{n} \frac{T_t}{(1+r)^t} = \sum_{t=0}^{n} \frac{g_t}{(1+r)^t} = G$$

  - Saving in a two-period model
    - Private saving: $s_1^p = w_1 - T_1 - c_1$
    - Public saving: $s_1^G = T_1 - g_1$
    - Total saving: $s^T = w_1 - T_1 - c_1 + T_1 - g_1 = w_1 - c_1 - g_1$

- Altruism
  - Live for 2 periods with 1 child

$$u(c_1, c_2) + v(c_1', c_2')$$

- Consumption taxes: $\tau_1, \dots, \tau_n$
  - Budget constraint

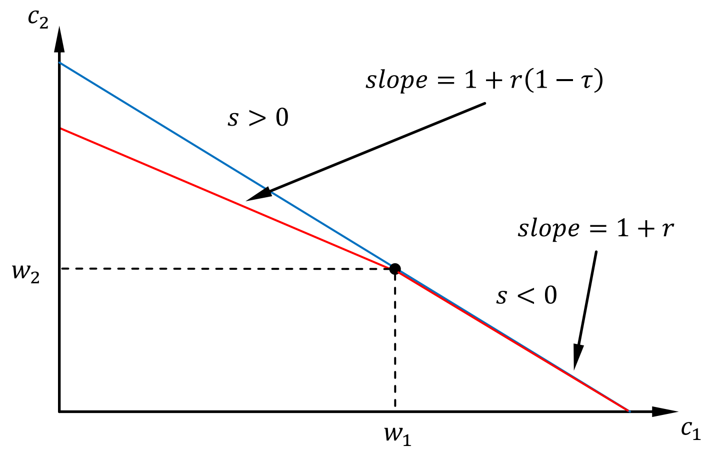
$$\sum_{t=0}^{n} \frac{w_t}{(1+r)^t} = \sum_{t=0}^{n} \frac{(1+\tau_t)c_t}{(1+r)^t}$$

    - If $\tau_1 = \tau_2 = \cdots = \tau_n$, then we have lump-sum taxes
    - If $\tau_i \neq \tau_j$, then we have substitution effect
  - Savings (in two-period model)
    - Private saving: $s^p = w_1 - (1+\tau_1)c_1$
    - Public saving: $s^G = \tau_1 c_1 - g_1$
    - Total saving: $s^T = w_1 - c_1 - g_1$

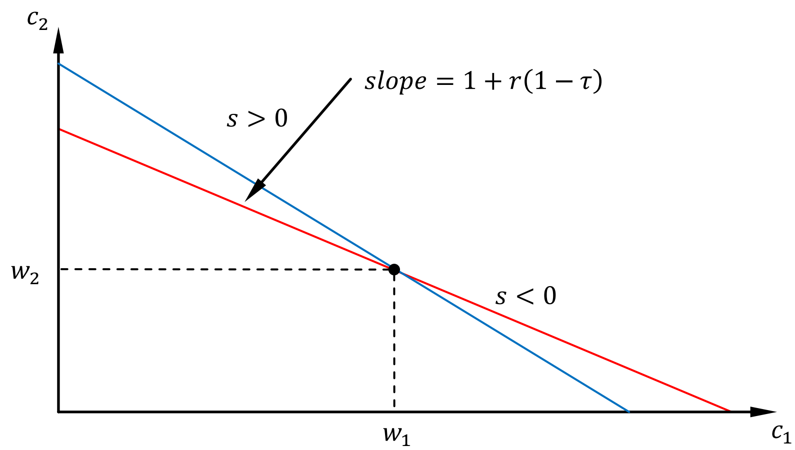
- Capital income taxes: τ (for all periods)
  - Budget constraint (for a given period t)

$$\begin{aligned} s_t + c_t &= s_{t-1}(1+r) - \tau s_{t-1} r + w_t \\ &= s_{t-1}[1 + r(1-\tau)] + w_t \end{aligned}$$

    - Note that if $s_t < 0$, the government is <u>subsidizing</u> borrowing
  - How to treat $s_t < 0$?
    - One way is to have no taxes (subsidy)



      - If $s < 0$, there is no effect
    - The other is to have subsidy

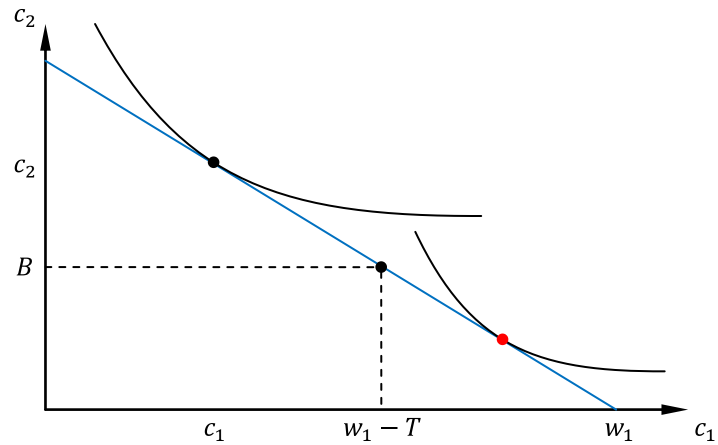


- If $s < 0$, substitution effect is $c_1 \uparrow$ and $c_2 \downarrow$; income effect is $c_1 \uparrow$ and $c_2 \uparrow$; overall, borrowing will increase

Taxation and Pension

- Question for today: why do we observe public pension program?

- Environment:
  - Same model with two periods, $w_1 > 0$ and $w_2 = 0$
  - Define T as the lump-sum tax in period 1, and B is benefit in period 2
  - Pension program is ***fully funded***, i.e. $B = (1+r)T$
  - Agent's budget constraint: $c_1 + \frac{1}{1+r}c_2 = w - T + \frac{B}{1+r}$



  - Effect of taxation:

	$B = 0$	$B > 0$
Private Saving	$w_1 - c_1$	$w_1 - c_1 - T$
Public Saving	0	T
Total	$w_1 - c_1$	$w_1 - c_1$

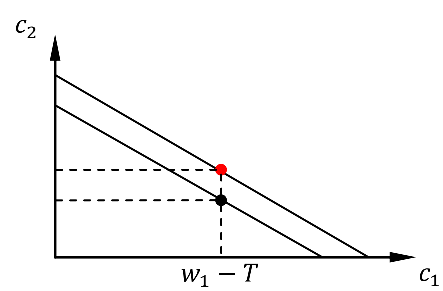
Thus, there is a complete crowd-out effect of public saving on private saving.

  - If financial markets are incomplete, then people with preference for current consumption (e.g. the red dot) may face borrowing constraints in the first period.

- Reasons for public pension system
  - $r_G > r$, i.e. the government is "more efficient" in saving

$$B = (1 + r_G)T \;\Rightarrow\; c_1 + \frac{c_2}{1+r} = w_1 + \frac{1+r_G}{1+r}T - T$$
$$= w_1 + \frac{r_G - r}{1+r}T$$

It used to be the case that governments have access to foreign financial markets, and hence better able to get a more diversified portfolio and make higher returns on investments.



  - The government is bigger, so it's better able to diversify risks.
  - But now financial markets are more efficient, and so this argument is not as strong as when

public pension was first introduced.
- The pay-as-you-go pension system, the argument is similar, with r_G replaced by n the population growth rate.

➢ Pay-as-you-go pension system (with overlapping generations)
- Denote n as the population growth rate. The population size is $N_t = (1+n)N_{t-1}$
- Government budget constraint:
$$N_t B = N_{t+1} T = (1+n) N_t T \;\;\Rightarrow\;\; B = (1+n)T$$
- Individual budget constraint:
$$c_1 + \frac{1}{1+r} c_2 = w_1 - T + \frac{1+n}{1+r} T = w + \frac{n-r}{1+r} T$$
- Therefore, we should have a pay-as-you-go pension system if the population grows faster, i.e. when $n > r$
- Also, note that wages grow over time. Consider a tax system that uses wage taxes (instead of lump-sum) to finance the pension system. The government budget constraint is:
$$\left.\begin{aligned} N_{t+1} &= (1+n)N_t \\ w_{t+1} &= (1+\lambda) w_t \end{aligned}\right\} \;\Rightarrow\;\; N_t B = t w_t (1+\lambda) N_t (1+n)$$
$$\Rightarrow\;\; B = t(1+\lambda)(1+n) w_t$$

➢ Insurance against death.
- The probability of death in the second period is ρ
- Government budget constraint in this case:
$$N_t (1-\rho) B = N_{t+1} T = (1+n) N_t T \;\;\Rightarrow\;\; B = \frac{1+n}{1-\rho} T$$
- Agent's budget constraint:
$$\begin{aligned} c_1 + \frac{c_2}{1+r} &= w_1 - T + \frac{1+n}{(1-\rho)(1+r)} T \\ &= w_1 + \frac{(1+n) - (1-\rho)(1+r)}{(1-\rho)(1+r)} T \end{aligned}$$

If $n = r$, so that fully funded system is the same as the pay-as-you-go system, then
$$\begin{aligned} c_1 + \frac{c_2}{1+r} &= w_1 + \frac{(1+r) - (1-\rho)(1+r)}{(1-\rho)(1+r)} T \\ &= w_1 + \frac{\rho}{1-\rho} T \end{aligned}$$
  - If $\rho \to 0$, then the situation is the same as in a fully funded system.
  - If $\rho \to 1$, then only a very small number of people can survive to the next period, and thus get to enjoy the benefit from everybody else who didn't make it.

➢ At the end of the day, a public pension system is there for distributive reasons. The government taxes people proportional to their income, but gives everybody the same amount of pension regardless of income.

Taxation and Risk Taking

- Domar-Musgrave principle.
  - One agent with wealth S to save
  - Two assets, one safe and one risky
    - Safe asset has return $1 + r_0$
    - Risky asset has return $1 + x$, where x is a random with mean μ and variance σ^2
  - Agent chooses to invest αS in the risky asset (we're interested in how the α changes when there is a tax system).
  - Period 2's income:
$$\frac{y}{S} = (1-\alpha)(1+r_0) + \alpha(1+x)$$
  - Expected utility: $Eu(y/S)$, which depends on μ and σ^2 (risk aversion)
    - Assume that
$$Eu\left(\frac{y}{S}\right) = E\left[\frac{y}{S}\right] - \frac{\lambda}{2}\text{Var}\left(\frac{y}{S}\right)$$
    Note that λ is the coefficient of risk aversion.
$$E\left(\frac{y}{S}\right) = (1-\alpha)(1+r_0) + \alpha(1+\mu) = 1 + (1-\alpha)r_0 + \alpha\mu$$
$$\text{Var}\left(\frac{Y}{S}\right) = (1-\alpha)^2 0 + \alpha^2\sigma^2$$
$$Eu\left(\frac{y}{S}\right) = 1 + (1-\alpha)r_0 + \alpha\mu - \frac{\lambda}{2}\alpha^2\sigma^2$$
    FOC with respect to α:
$$\mu - r_0 = x\alpha\sigma^2 \;\;\Rightarrow\;\; \alpha = \frac{u - r_0}{\lambda\sigma^2}$$
  - If $r_0 = 0$ and full loss deductibility (i.e. government fully subsidizes the loss in investments)
$$\frac{y}{S} = 1 + (1-\alpha)0(1-t) + \alpha x(1-t)$$
$$\Rightarrow\;\; E\left(\frac{y}{S}\right) = 1 + (1-t)\alpha\mu, \qquad \text{Var}\left(\frac{y}{S}\right) = \alpha^2(1-t)^2\sigma^2$$
  - Agent's problem:
$$\max_{\alpha} 1 + (1-t)\alpha\mu - \frac{\lambda}{2}\alpha^2(1-t)^2\sigma^2$$
$$\Rightarrow\;\; (1-t)\mu = \lambda\alpha(1-t)^2\sigma^2$$
$$\Rightarrow\;\; \alpha = \frac{\mu}{\lambda(1-t)\sigma^2}$$
    - Extensions: try the problem with $r_0 \neq 0$ and the case where the government does not have full loss offset.

Taxation and Risk Taking (cont'd)

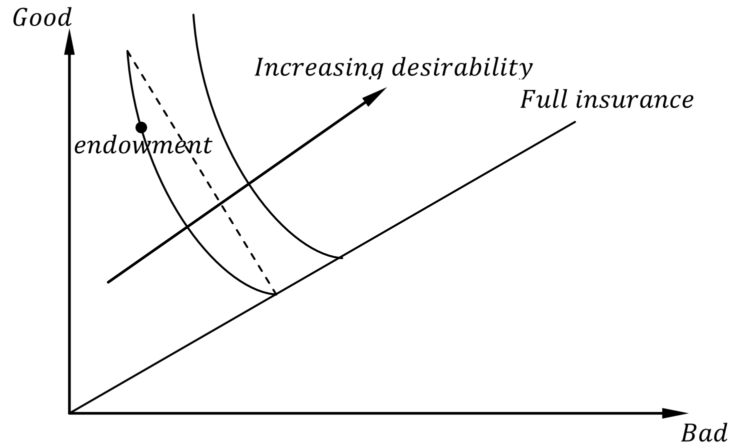
❖ Recall from last lecture:

$$\mathrm{E}\,u\left(\frac{y}{S}\right) = \mathrm{E}\left[\frac{y}{S}\right] - \frac{\lambda}{2}\mathrm{Var}\left(\frac{y}{S}\right), \qquad \text{where } \frac{y}{S} = 1 + (1-\alpha)r_0 + \alpha x$$

$$= 1 + (1-\alpha)r_0 + \alpha\mu - \frac{\lambda}{2}\alpha^2\sigma^2$$

FOC with respect to α is

$$\mu - r_0 - \lambda\alpha\sigma^2 = 0 \ \Rightarrow\ \alpha^* = \frac{\mu - r_0}{\lambda\sigma^2}$$



❖ With full loss offset

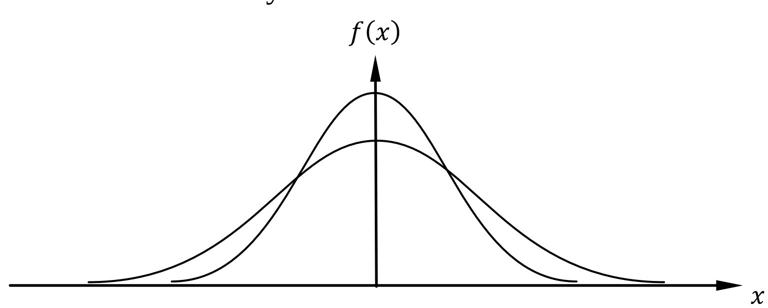
$$\frac{y}{S} = 1 + (1-\alpha)r_0(1-t) + \alpha x(1-t)$$

$$\mathrm{E}\,u = 1 + (1-\alpha)r_0(1-t) + \alpha\mu(1-t) - \frac{\lambda}{2}\alpha^2(1-t)^2\sigma^2$$

FOC w.r.t α is

$$\underbrace{(\mu - r_0)(1-t)}_{\substack{\text{marginal benefit}\\ \text{of risky asset}}} = \underbrace{\lambda\alpha(1-t)^2\sigma^2}_{\substack{\text{marginal cost}\\ \text{of risky asset}}} \ \Rightarrow\ \alpha = \frac{\mu - r_0}{\lambda(1-t)\sigma^2}$$

- If $t\uparrow$, then $\alpha\uparrow$.
- Taxing capital income actually decreases the riskiness of investment: $(1-t)\sigma^2 < \sigma^2$. So taxation. So when people are risk averse, they invest more when there is capital income tax because investment is less risky now.



- Government revenue is $S\alpha t x + S(1-\alpha)t\, r_0$

- With no loss offset

$$\frac{y}{S} = \begin{cases} 1 + (1-\alpha) r_0 (1-t) + \alpha x (1-t) & \text{if } x \geq 0 \\ 1 + (1-\alpha) r_0 (1-t) + \alpha x & \text{if } x < 0 \end{cases}$$

  - Define $\mathrm{E}[x|x > 0]$ as the condition average where $x \geq 0$
  - Define $\mathrm{Var}[x|x \geq 0]$ as the condition variance

  Then,

$$\begin{aligned} \mathrm{E}\, u = 1 + (1-\alpha) r_0 (1-t) \\ &+ \Pr(x \geq 0) \left[\alpha\, \mathrm{E}[x|x \geq 0]\,(1-t) - \frac{\lambda}{2} \alpha^2 (1-t)^2\, \mathrm{Var}(x|x \geq 0) \right] \\ &+ \Pr(x < 0) \left[\alpha\, \mathrm{E}[x|x < 0] - \frac{\lambda}{2} \alpha^2\, \mathrm{Var}(x|x \geq 0) \right] \end{aligned}$$

  The FOC is

$$\begin{aligned} -r_0(1-t) + \Pr(x \geq 0)\, \mathrm{E}[x|x \geq 0]\,(1-t) + \Pr(x < 0)\, \mathrm{E}[x|x < 0] \\ - \Pr(x \geq 0)\, \lambda\alpha(1-t)^2\, \mathrm{Var}(x|x \geq 0) - \Pr(x < 0)\, [\lambda\alpha\, \mathrm{Var}(x|x < 0)] = 0 \end{aligned}$$

$$\begin{aligned} [\Pr(x \geq 0)\, \mathrm{E}(x|x \geq 0) - r_0](1-t) + \Pr(x < 0)\, \mathrm{E}[x|x < 0] \\ - \Pr(x \geq 0)\, \lambda\alpha(1-t)^2\, \mathrm{Var}(x|x \geq 0) - \Pr(x < 0)\, \lambda\alpha\, \mathrm{Var}(x|x < 0) = 0 \end{aligned}$$

Human Capital Investment

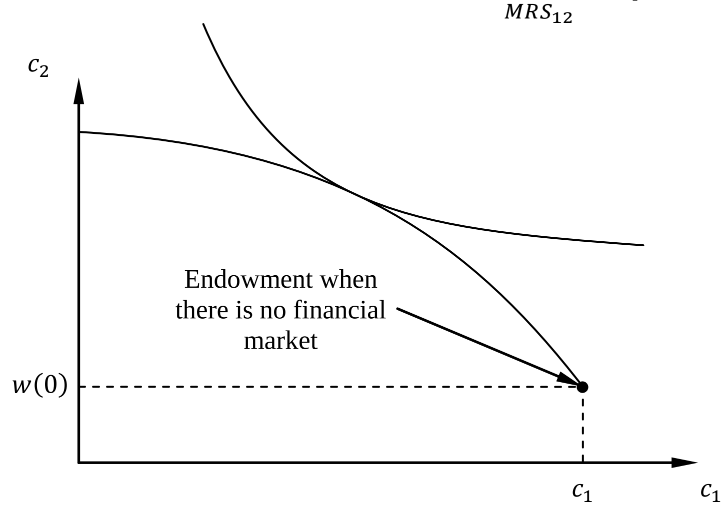
- Consider a 2-period model with one agent, whose utility is $u(c_1, c_2) = u(c_1) + \delta u(c_2)$
  - Period 1:
    - Start with income m
    - Can consume c_1 or invest in human capital h
    - With no financial market, budget constraint is $m = c_1 + h$
    - With financial market, budget constraint is $m = c_1 + h + s$
  - Period 2 (assume labor supply is inelastic):
    - Get wage $w(h)$, with $w(0) > 0$, $w'(h) > 0$ and $w''(h) < 0$
    - With no financial market, budget constraint is $c_2 = w(h)$
    - With financial market, budget constraint is $c_2 = w(h) + (1+r)s$

  - Agent's problem (with no financial market)

$$\max_{c_1,c_2,h} u(c_1) + \delta u(c_2), \qquad s.t. \begin{cases} m = c_1 + h \\ c_2 = w(h) \end{cases}$$
$$\Leftrightarrow \max_h u(m-h) + \delta u\big(w(h)\big)$$

The FOC is

$$u'(m-h) = \delta u'\big(w(h)\big)w'(h) \;\; \Rightarrow \;\; \underbrace{\frac{u'(c_1)}{\delta u'(c_2)}}_{MRS_{12}} = \underbrace{w'(h)}_{\text{price ratio}}$$

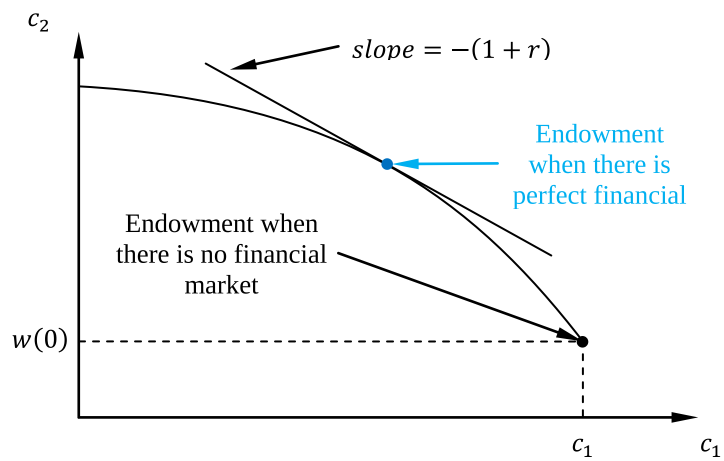


  - Agent's problem (with perfect financial market)

$$\max_{c_1,c_2,h,s} u(c_1) + \delta u(c_2), \qquad s.t. \begin{cases} m = c_1 + h + s \\ c_2 = w(h) + s(1+r) \end{cases}$$
$$\Leftrightarrow \max_{h,s} u(m-h-s) + \delta u\big(w(h) + s(1+r)\big)$$

The FOC's are

$$\left.\begin{array}{ll} h: & -u'(c_1) + \delta u'(c_2)w'(h) = 0 \\ s: & -u'(c_1) + \delta u'(c_2)(1+r) = 0 \end{array}\right\} \Rightarrow \boxed{w'(h) = 1+r}$$
$$\Rightarrow \frac{u'(c_1)}{\delta u'(c_2)} = 1 + r = w'(h)$$

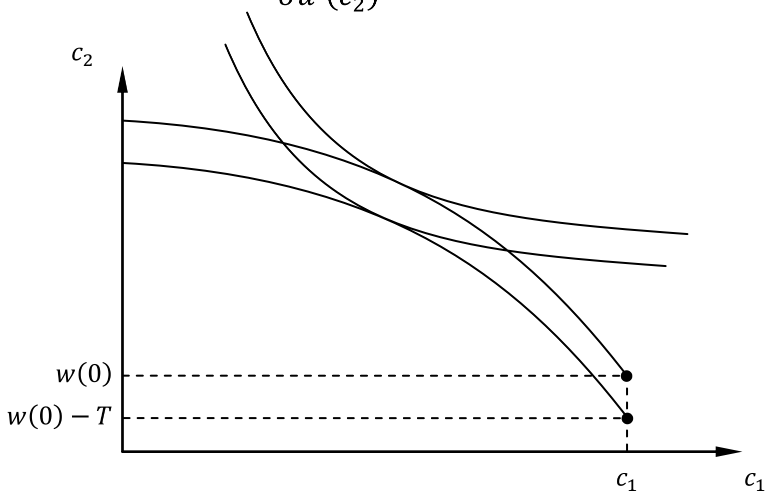


- Effect of taxes on human capital investment
  - Suppose there is lump-sum wealth tax T and no financial market

$$\max_h u(m - T - h) + \delta u\big(w(h)\big)$$

FOC is

$$\frac{u'(c_1)}{\delta u'(c_2)} = w'(h)$$



  - Consider a uniform tax t on consumption

$$\max_h u\left(\frac{1}{1+t}(m-h)\right) + \delta u\left(\frac{1}{1+t}w(h)\right)$$

where

$$(1+t)c_1 = m - h, \qquad (1+t)c_2 = w(h)$$

The FOC is still

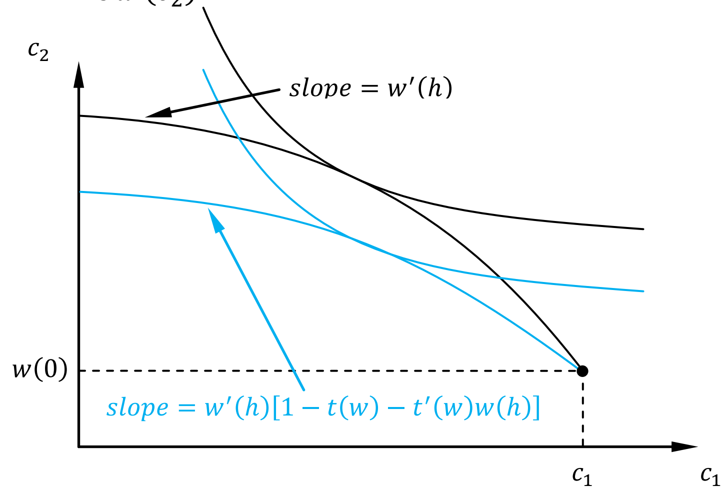
$$\frac{u'(c_1)}{\delta u'(c_2)} = w'(h)$$

- Progressive income tax, $t(w)$ with $t'(w) > 0$

$$\max_h u(m-h) + \delta u\big(w(h)[1 - t\big(w(h)\big)]\big)$$

The FOC is

$$\frac{u'(c_1)}{\delta u'(c_2)} = w'(h)[1 - t(w) - t'(w)w(h)]$$



- In this case, there is distortion.

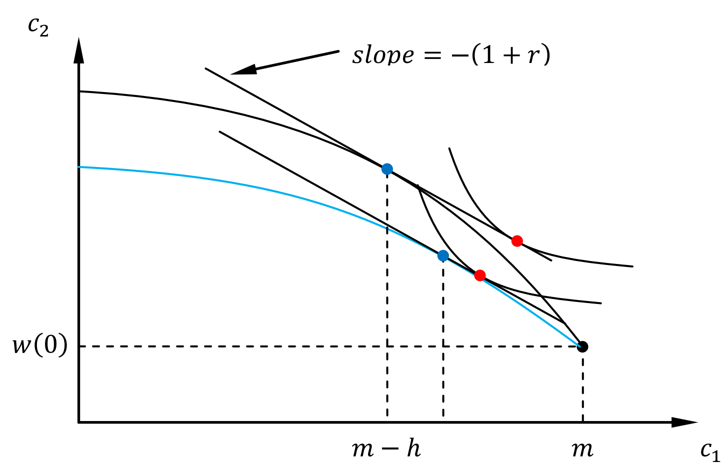
- Progressive tax when there is perfect financial market

$$\max_{h,s} u(m-h-s) + \delta u\left(w(h)\big[1 - t\big(w(h)\big)\big] + s(1+r)\right)$$

The FOC's are

$$h: \quad -u'(c_1) + \delta u'(c_2) \cdot w'(h)\big[1 - t\big(w(h)\big) - t'\big(w(h)\big)w(h)\big] = 0$$
$$s: \quad -u'(c_1) + \delta u'(c_2)(1+r) = 0$$
$$\Rightarrow \quad w'(h)[1 - t(\cdot) - t'(\cdot)w(h)] = (1+r)$$
$$\Rightarrow \quad \frac{u'(c_1)}{\delta u'(c_2)} = 1 + r$$

- Note that in this case, investment in human capital is distorted: one will under-invest in human capital because its return will be taxed.
- However, consumption is not distorted, because one can use the alternative



- This is an example of taxing one market, and distorting only one market.
- If we tax savings instead, however, then both markets will be distorted, because

$$\frac{u'(c_1)}{\delta u'(c_2)} = w'(h) = (1-t)(1+r)$$

Marginal Cost of Public Fund

- Public good: non-rival and non-excludable
- Suppose individual utility is
$$u^i(x_i, G) = x_i + v(G)$$
where x_i is the private consumption for i, and G is the total provision of public good.
  - Public good is financed through lump-sum tax T
  - Then agent's problem is
$$\max_{x_i} x_i + v(G), \qquad s.t.\ x_i = m - T$$
Hence we derive the indirect utility
$$V^i(T, G) = m - T + v(G)$$
  - The government's budget constraint is $NT = cG$, where N is the number of agents and c is the marginal cost of public good. The government's problem is
$$\max_{T,G} \sum_{i=1}^{N} V^i(T, G), \qquad s.t.\ NT = cG \ \Leftrightarrow\ \max_G N\left[m - \frac{cG}{N} + v(G)\right]$$
The FOC is
$$\underbrace{Nv'(G)}_{\substack{\text{sum of private}\\ \text{marginal benefit}}} = \underbrace{c}_{\substack{\text{marginal}\\ \text{public cost}}}$$

- Distortionary Taxes
  - Denote the base of tax as $B(t)$, with $B'(t) < 0$
    - Examples: wage tax, investment tax, consumption tax
  - Agent's problem:
$$\max_x x + v(G), \qquad s.t.\ x = B(t)(1 - t)$$
$$\Rightarrow\ V(t, G) = B(t)(1 - t) + v(G)$$
  - Government budget constraint: $NB(t)t = cG$. The problem of the government is
$$\max_{t,G} \sum_{i=1}^{n} B(t)(1 - t) + v(G), \qquad s.t.\ B(t)t = \frac{cG}{N}$$

Marginal Cost of Public Fund (cont'd)

❖ ***Samuelson rule*** (with lump-sum tax):

$$\sum_{i=1}^{N} \underbrace{v'(G)}_{\substack{\text{private}\\ \text{marginal}\\ \text{benefit}}} = \underbrace{c}_{\substack{\text{MC of}\\ \text{public}\\ \text{good}}}$$

❖ Distortionary taxes (tax base is $B(t)$, and $B'(t) < 0$)

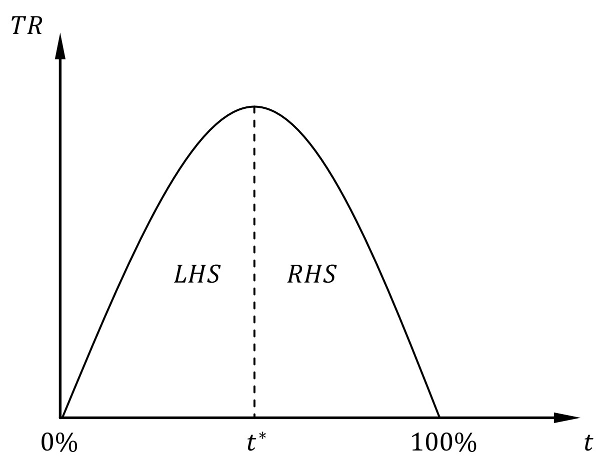
- Tax revenue: $NtB(t)$

$$\frac{\partial TR}{\partial t} = N\left[\underbrace{B(t)}_{\oplus} + \underbrace{tB'(t)}_{\ominus}\right] = N\,B(t)\left[1 + \frac{tB'(t)}{B(t)}\right]$$
$$= N\,B(t)\left[1 - \epsilon_{B,t}\right]$$

where $\epsilon_{B,t} = -\,tB'(t)/B(t)$ is the elasticity of the tax base with respect to t.

$$\frac{\partial TR}{\partial t} = N\,B(t)\left[1 - \epsilon_{B,t}\right] > 0, \qquad \text{if } \epsilon_{B,t} < 1$$

- Laffer curve



- Government budget constraint: $NtB(t) = cG$

❖ Agent's problem:

$$\max x_i + v(G), \qquad s.t.\ x_i = B(t)(1-t)$$

❖ Government's problem:

$$\max_{G,t} \sum_{i=1}^{N} B(t)(1-t) + \sum_{i=1}^{N} v(G), \qquad s.t.\ G = \frac{NB(t)t}{c}$$
$$\Rightarrow \max_{t} NB(t)(1-t) + Nv\left(\frac{NB(t)t}{c}\right)$$

FOC:

$$NB'(t)(1-t) - NB(t) + \frac{N^2}{c} v'(G)[B(t) + B'(t)t] = 0$$

$$\Rightarrow \quad B'(t) - [B'(t)t + B(t)] + \frac{N}{c} v'(G)[B(t) + B'(t)t] = 0$$

$$\Rightarrow \quad Nv'(G) \underbrace{[B(t) + B'(t)t]}_{=1-\epsilon_{Bt}} = \left\{ \underbrace{[B'(t)t + B(t)]}_{=1-\epsilon_{Bt}} - B'(t) \right\} c$$

$$\Rightarrow \quad \boxed{Nv'(G) = \underbrace{\left[1 - \frac{B'(t)}{1 - \epsilon_{Bt}}\right]}_{\oplus} c}$$

- This is the "Samuelson rule" for the case of distortionary taxes
- Note that $B'(t) < 0$, and $\epsilon_{Bt} < 1$, and so

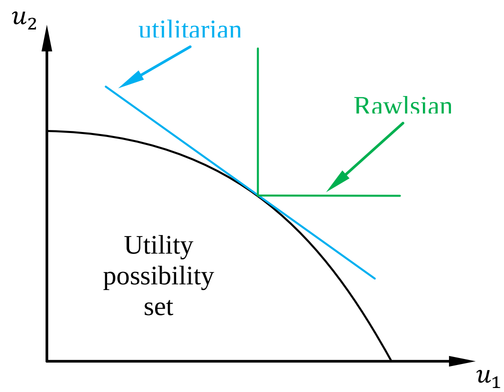
$$1 - \frac{B'(t)}{1 - \epsilon_{Bt}} > 1$$

  - If $\epsilon_{Bt} > 1$, the government can reduce tax and increase the provision of public good (as well as increase the tax base $B(t)$).
  - This is also our ***marginal cost of public fund* (*MCPF*)**:

$$1 - \frac{B'(t)}{1 - \epsilon_{Bt}} = 1 + \frac{B(t)}{t} \cdot \frac{\epsilon_{Bt}}{1 - \epsilon_{Bt}}$$

    - High elasticity $\rightarrow$ large MCPF
      - MCPF in Canada is estimated to be around 1.5 to 2

Social Welfare Function



- ***Utilitarian social welfare function***

$$U(u_1, \dots, u_n) = \sum_{i=1}^{n} u_i(x_i)$$

- ***Rawlsian social welfare function***

$$U(u_1, \dots, u_n) = \min\{u_1(x_1), \dots, u_n(x_n)\}$$

- Suppose $w_1 > w_2$, and the government wants to transfer T from agent 1 to agent 2. The agents' utility functions are

$$u(w_1 - T), \qquad \lambda u(w_2 + T)$$

where $\lambda \in [0,1]$.
    - The utilitarian's problem

$$\max_T u(w_1 - T) + \lambda u(w_2 + T) \;\; \Rightarrow \;\; u'(w_1 - T) = \lambda u'(w_2 + T)$$

        - The key here is to choose T to equalize *marginal* utilities
    - The Rawlsian's problem

$$\max_T \min\{u(w_1 - T), \lambda u(w_2 + T)\} \;\; \Rightarrow \;\; u(w_1 - T) = \lambda u(w_2 + T)$$

        - The key here is to choose T to equalize utilities

- ***Generalized social welfare function*** (with aversion to inequality)

$$W = \sum_{i=1}^{n} \frac{u_i^{1-\rho} - 1}{1 - \rho}$$

    - If $\rho = 0$, we have the utilitarian SWF
    - If $\rho \to 1$, we have the Rawlsian SWF

Optimal Income Taxation

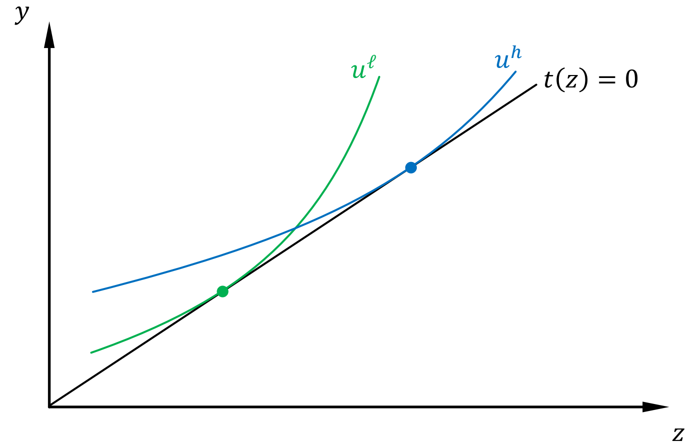
- 2 types of agents, high or low. Agents choose labor supply L
- Wages are fixed: $w^h = \eta^h$ for high type, and $w^\ell = 1 - \eta^h$ for low type
- Government cannot observe w^i and L, but can observe total income $z = w^i L$
- Income taxes: $t(z)$, with the marginal tax rate $t'(z)$ and the average tax rate $ATR = t(z)/z$
- There is one consumption good y with price one
- Agent's budget constraint:

$$y = w^i L - t(z) = z - t(z)$$

- Preference: $u(y, L)$ with $u_y > 0$, $u_{yy} \leq 0$, $u_L < 0$, and $u_{LL} \geq 0$
  - Rewrite preference as $u\left(y, z/w^i\right)$

$$du\left(y, \frac{z}{w^i}\right) = u_y(\cdot)dy + \frac{u_L(\cdot)}{w^i}dz \;\;\Rightarrow\;\; \left.\frac{dy}{dz}\right|_{\bar{u}} = -\frac{1}{w^i} \cdot \frac{u_L(\cdot)}{u_y(\cdot)} > 0$$

    With appropriate assumption, we can get $\frac{\partial^2 y}{\partial z^2} > 0$.
  - With no taxes, the economy is described by the following.



Optimal Income Taxation (cont'd)

- Environment :
  - Utility: $u\left(y, \frac{z}{w_i}\right)$

$$\left.\frac{\partial y}{\partial z}\right|_{\bar{u}} = -\underbrace{\frac{u_L(\cdot)}{u_y(\cdot)}}_{\substack{\text{MRS} \\ \text{between} \\ L \text{ and } y}} \cdot \underbrace{\frac{1}{w_i}}_{\substack{\text{larger wage} \\ \text{implies lower} \\ \text{hours to get} \\ \text{same income}}} > 0$$

Assume that $u_{Ly}(\cdot) = 0$, so that utility is separable,

$$\left.\frac{\partial^2 y}{\partial z^2}\right|_{\bar{u}} = -\frac{1}{w_i}\left[\frac{u_{LL}(\cdot)u_y(\cdot) - u_{yy}(\cdot)u_L(\cdot)}{\left(u_y(\cdot)\right)^2}\right] > 0$$

Budget constraint:

$$y = wL - T = z - t(z)$$

If $t(z) = 0$, then $y = z$.

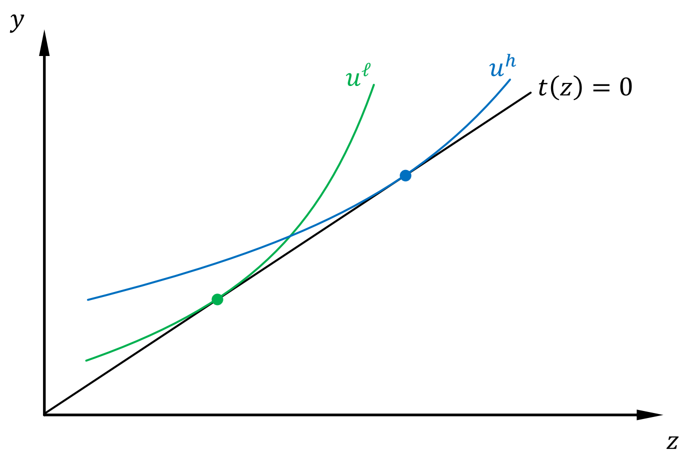
The tangencies are the solution to the following maximization problem:

$$\max_{y,z} u\left(y, \frac{z}{w_i}\right), \qquad s.t.\ y = z$$

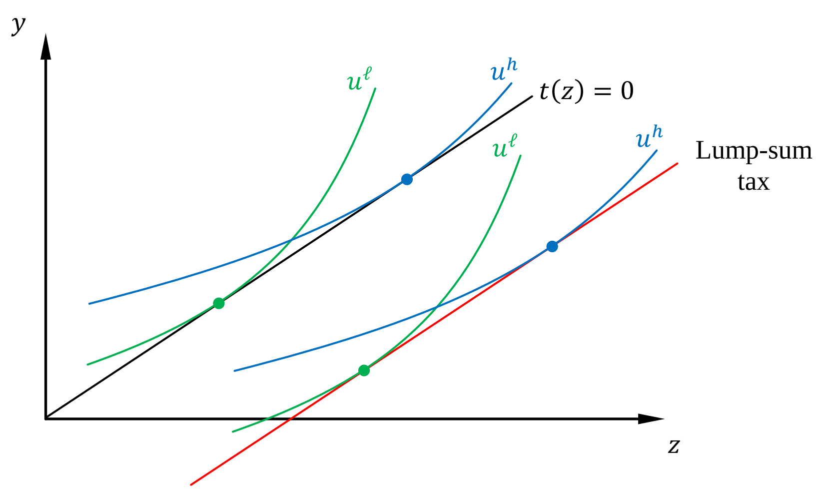
The FOC is

$$u_y(\cdot) + \frac{1}{w_i}u_L(\cdot) = 1$$

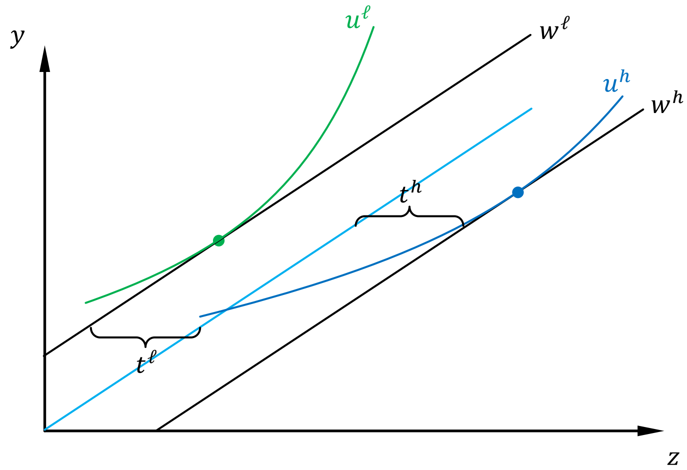
$$\Leftrightarrow \ -\frac{u_L(\cdot)}{u_y(\cdot)w_i} = 1$$



- In the case of a lump-sum tax for both types, the graph looks like



- Suppose w was observable and set $t^h > 0$ and $t^\ell < 0$ as lump-sum taxes



For the low type,

$$\max_z u\left(z - t^\ell, \frac{z}{w^\ell}\right) \;\Rightarrow\; -\frac{u_L}{u_y w^\ell} = 1$$

  - Note that the high type has incentive to imitate the low type, given the way the graph is drawn.

- First best solution: t^ℓ and t^h, $y_i = z - t^i$
- Government budget constraint:

$$n^h[z^h - y^h] + n^\ell[z^\ell - y^\ell] = G$$

- Utilitarian solution:

$$\max_{z^h, z^\ell, y^h, y^\ell} n^h u\left(y^h, \frac{z^h}{w^h}\right) + n^\ell u\left(y^\ell, \frac{z^\ell}{w^\ell}\right) + \lambda\left[n^h[z^h - y^h] + n^\ell[z^\ell - y^\ell] - G\right]$$

At the optimum,

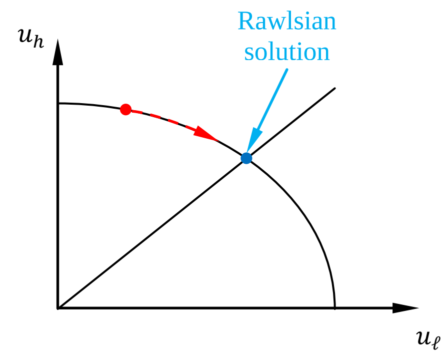
$$-\frac{u_L(\cdot)}{u_y(\cdot)} \cdot \frac{1}{w^h} = 1, \qquad -\frac{u_L(\cdot)}{u_y(\cdot)} \cdot \frac{1}{w^\ell} = 1$$

- What happens if the government sets $t(z)$, since only z is observable?
  - Note that $y_i = w_i L - t(z)$

$$y = z - t(z) \;\Rightarrow\; \frac{\partial y}{\partial z} = 1 - t'(z)$$

Optimal Income Taxation (cont'd)

- Recall from last time...
  - Individual's budget constraint:
$$y_i = z_i - t(z_i)$$
    and
$$t'(z_i) = \text{marginal tax rate}$$
$$\frac{t(z)}{z} = \text{average tax rate}$$
    Thus,
$$\frac{\partial y}{\partial z} = 1 - t'(z)$$
  - Government's budget constraint:
$$n_h[z_h - y_h] + n_\ell[z_\ell - y_\ell] = G$$
    where $t(z_i) = z_i - y_i$ is from the individuals' budget constraint.
  - The utilitarian government solves
$$\max_{z_h, z_\ell, y_h, y_\ell} n_n u\left(y_h, \frac{z_h}{w_h}\right) + n_\ell u\left(y_\ell, \frac{z_\ell}{w_\ell}\right) + \lambda[n_h[z_h - y_h] + n_\ell[z_\ell - y_\ell] - G]$$
    with FOCs
$$\left.\begin{array}{ll} z_h: & n_h u_L(\cdot)\dfrac{1}{w_h} + \lambda n_h = 0 \\ y_h: & n_h u_y(\cdot) - \lambda n_h = 0 \end{array}\right\} \Rightarrow \underbrace{-\frac{u_L(\cdot)}{u_y(\cdot)}}_{MRS_{Ly}} = 1 = \left(1 - t'(z)\big|_{t'(z)=0}\right)$$
    - Here, the marginal tax rate $t'(z) = 0$ because tax is lump-sum. This is also optimal, as there is no distortion. The average tax rate depends on how much the government redistributes between the two groups, and the functional form of the utility function.
    - In terms of the Pareto frontier, redistribution will lead to movement along the Pareto frontier towards the centre. But it won't necessarily stop at the mid-point (or 45 degree line). If the government is Rawlsian, the solution will be at the mid-point; whereas a utilitarian government's solution depends on the function form. For example, if utility function is $u(y) - L$, then the solution will be passed the midpoint.



- Asymmetric Information between government and agents
  - Self selection constraint
$$u\left(y_h, \frac{z_h}{w_h}\right) \geq u\left(y_\ell, \frac{z_\ell}{w_h}\right)$$
  - Now the government's problem is
$$\max_{y_h, z_h, y_\ell, z_\ell} n_h u\left(y_h, \frac{z_h}{w_h}\right) + n_\ell u\left(y_\ell, \frac{z_\ell}{w_\ell}\right) + \lambda[n_h(z_h - y_h) + n_\ell(z_\ell - y_\ell) - G]$$
$$+ \psi\left[n_h u\left(y_h, \frac{z_h}{w_h}\right) - n_h u\left(y_\ell, \frac{z_\ell}{w_h}\right)\right]$$

with FOC for z_h:

$$n_h u_L\left(y_h, \frac{z_h}{w_h}\right)\frac{1}{w_h} + \lambda n_h + \psi n_h u_L\left(y_h, \frac{z_h}{w_h}\right)\frac{1}{w_h} = 0 \;\Rightarrow\; u_L\left(y_h, \frac{z_h}{w_h}\right)\frac{1}{w_h} = -\frac{\lambda}{1+\psi}$$

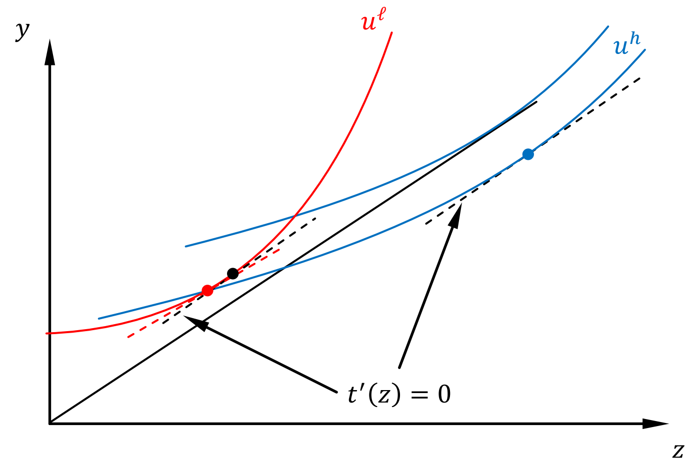
FOC for y_h:

$$n_h u_y\left(y_h, \frac{z_h}{w_h}\right) - \lambda n_h + \psi n_h u_y\left(y_h, \frac{z_h}{w_h}\right) = 0 \;\Rightarrow\; u_y\left(y_h, \frac{z_h}{w_h}\right) = \frac{\lambda}{1+\psi}$$

Combining the two yields

$$-\frac{u_L\left(y_h, \frac{z_h}{w_h}\right)}{u_y\left(y_h, \frac{z_h}{w_h}\right)}\frac{1}{w_h} = 1 = 1 - t'(z) \;\Rightarrow\; t'(z) = 0$$

This means that the marginal tax rate for the rich is going to be zero. In other words, we have <u>no distortion at the top</u>!



FOC for z_ℓ:

$$n_\ell u_L\left(y_\ell, \frac{z_\ell}{w_\ell}\right)\frac{1}{w_\ell} + \lambda n_\ell - \psi n_h u_L\left(y_\ell, \frac{z_\ell}{w_h}\right)\frac{1}{w_h} = 0$$

FOC for y_ℓ:

$$n_\ell u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right) - \lambda n_\ell - \psi n_h u_y\left(y_\ell, \frac{z_\ell}{w_h}\right) = 0$$

From the two conditions,

$$u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right) - \psi\frac{n_h}{n_\ell}u_y\left(y_\ell, \frac{z_\ell}{w_h}\right) = \lambda = -u_L\left(y_\ell, \frac{z_\ell}{w_\ell}\right)\frac{1}{w_\ell} + \psi\frac{n_h}{n_\ell}u_L\left(y_\ell, \frac{z_\ell}{w_h}\right)\frac{1}{w_h}$$

Divide both sides by $u_y(\cdot_\ell)$:

$$1 - \psi\frac{n_h}{n_\ell}\left[\frac{u_y\left(y_\ell, \frac{z_\ell}{w_h}\right)}{u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right)} + \frac{u_L\left(y_\ell, \frac{z_\ell}{w_h}\right)}{u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right)}\frac{1}{w_h}\right] = \underbrace{-\frac{u_L\left(y_\ell, \frac{z_\ell}{w_\ell}\right)}{u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right)}\frac{1}{w_\ell}}_{MRS_{Ly}}$$

Since individuals always maximize utility, we have

$$\underbrace{-u_L\left(y_\ell, \frac{z_\ell}{w_\ell}\right)\frac{1}{w_\ell}}_{MRS_{Ly}} = 1 - t'(z)$$

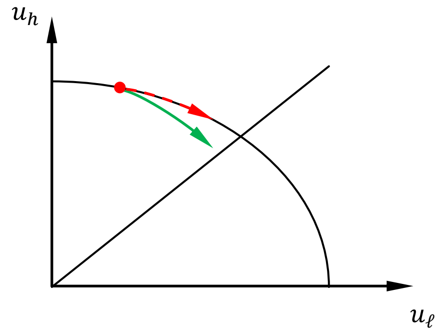
Thus,

$$t'(z) = \psi \frac{n_h}{n_\ell}\left[\frac{u_y\left(y_\ell, \frac{z_\ell}{w_h}\right)}{u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right)} + \frac{u_L\left(y_\ell, \frac{z_\ell}{w_h}\right)}{u_y\left(y_\ell, \frac{z_\ell}{w_\ell}\right)}\frac{1}{w_h}\right]$$

- Note that ψ measures how much the self selection constraint is binding

Optimal Income Taxation (cont'd)

- Wrapping up from last time...
  - The second best illustrated graphically



So in the second best world, we move into the feasible set, but no longer on the Pareto frontier

- Extensions to the basic model...
  - Add more types:
    - Different marginal tax rates for all but the top wage earners
    - Pooling for some types
    - No distortion at the high end, but distortion at the other levels may occur
  - Linear tax system only – adding restriction to the tax system
    - Flat tax and lump-sum benefit
    - "No distortion at the top" is no longer true, because no distortion at the top means to have zero tax for the top earners. But this is not feasible under a flat tax system.
  - Introducing a public good to the economy
    - The tax revenue is spent on both redistribution and the provision of public good
    - Utility function of the individual becomes

$$u(y, L, G) = u\left(y, \frac{z}{w_i}, G\right)$$

    - The issue here is to examine how does the problem good affect the self-selection constraint?
      - E.g. suppose the public good is complement to leisure. That's going to make mimickers of low income people less willing to do so because by working less, the potential high earners also pay less tax, resulting in lower level of public good provision, and hence the high earners enjoy their leisure less as leisure and the public good are complementary to each other.
      - Redistribution in kind, work fair, etc., are all means to relax the self-selection constraint. The idea is to hurt the mimicker more than the targeted group of people.
      - Imposing inefficiency in one sector might correct for the inefficiency in another sector

World of Multiple Governments

- Median voter theorem
  - Direct democracy: citizens voting for policies
  - Set of policies G
  - Any individual can propose policy $g \in G$ against the status quo g'
  - All vote on the proposal and 50% plus one vote wins
  - Solve for policy that can beat any other policy g'
  - The solution is that individual with the median preference wins

  - Restrictions
    - One dimensional policy space
    - Single-peaked preferences

  - Example.
    - Suppose there is a large number of individuals N with income w
    - Preference is $c + \alpha_i u(G)$, and α_m is the median α
    - Let a lump-sum tax T be imposed, so that $c = w - T$
    - Government budget constraint: $G = NT$
    - The dictator solution

$$\max_{G,T} c + \alpha_i u(G)\,, \qquad s.t. \begin{cases} c = w - T \\ G = NT \end{cases} \iff \max_{G} w - \frac{G}{N} + \alpha_i u(G)$$

FOC:

$$\alpha_i u'(G) = \frac{1}{N} \;\Rightarrow\; \alpha_i N u'(G) = 1 \;\Rightarrow\; \text{solution} = G_i$$

SOC:

$$\alpha_i u''(G) < 0$$

- Horizontal Tax Competition
  - Multiple governments (assume $N = 2$)
  - Mobile tax base
    - Income tax → migration
    - Sales tax → cross-border shopping
    - Capital tax → investments or firm location
  - Source based taxation
    - Taxes are paid where capital is employed

  - Consider two regions A, B
  - $\bar{K}$ unit of capital need to be invested in either A or B, $K_A + K_B = \bar{K}$
  - Return in each region is $F_i(K_i)$, with $F_i'(K_i) > 0$ and $F_i''(K_i) \leq 0$
  - Tax on <u>return to capital</u> t_i, so that tax revenue is $t_i K_i F_i'(K_i)$.
    - Net return is $(1 - t_i) K_i F_i'(K_i)$
  - Timing
    - Governments set t_A and t_B that maximizes some <u>objective function</u>, e.g.
      - Tax revenue
      - $F(K) - KF'(K)$

- $F(K,L) - KF'(K,L)$
- Add public goods into the objective function

- Capital owner invest K_A and K_B and get return net of taxes

- Case 1. $F_i'(K_i) = R$ so that $F_i''(K_i) = 0$
  - For any t_A, t_B,

$$K_A = \begin{cases} \bar{K} & \text{if } t_A < t_B \\ 0 & \text{if } t_A > t_B \\ \bar{K}/2 & \text{if } t_A = t_B \end{cases}$$

  - Region i maximizes $(1 - t_i)R$, the per unit return in A. This is like a Bertrand competition, and the optimal solution is $t_A = t_B = 0$.
- Case 2. $F_i'(K_i) > 0$ and $F_i''(K_i) < 0$
  - Allocation of K_A and $K_B = \bar{K} - K_A$
  - For given t_A, t_B
  - Per unit return in i is $(1 - t_i)F_i'(K_i)$
  - The arbitrage condition

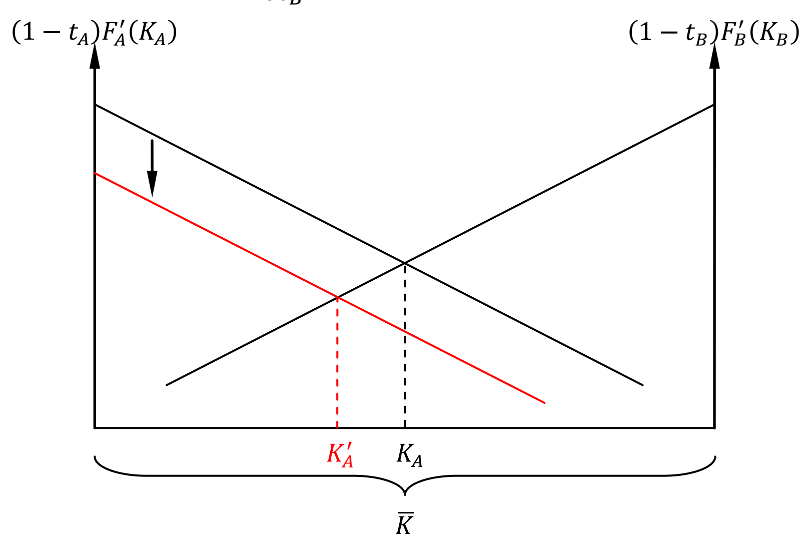
$$(1 - t_A)F_A'(K_A) = (1 - t_B)F_B'(\bar{K} - K_A) \ \Rightarrow\ \text{solution} = K_A(t_A, t_B)$$

Taking total differential will give

$$[(1 - t_A)F_A''(K_A) + (1 - t_B)F_B''(\bar{K} - K_A)]dK_A = F_A'(K_A)dt_A$$

$$\Rightarrow\ \frac{\partial K_A}{\partial t_A} = \frac{F_A'(K_A)}{[(1 - t_A)F_A''(K_A) + (1 - t_B)F_B''(\bar{K} - K_A)]} < 0$$

Similarly we can verify that $\frac{\partial K_A(t_A,t_B)}{\partial t_B} < 0$.



  - Region A's problem

$$\max_{t_A} t_A K(t_A, t_B)F'\big(K_A(t_A, t_B)\big)$$

FOC is

$$K_A(t_A,t_B)F'\big(K_A(t_A,t_B)\big) + t_A \frac{\partial}{\partial K_A(t_A,t_B)}\big[K_A(t_A,t_B)F''\big(K_A(t_A,t_B)\big) + F'\big(K_A(t_A,t_B)\big)\big] \cdot \frac{\partial K_A(t_A,t_B)}{\partial t_A}$$

Solve for $t_A(t_B)$. This is the reaction function.

- Can verify that $t_A'(t_B) > 0$. This means that t_A and t_B are strategic complements. The higher t_B, the more capital is flowing into region A, thus the less elastic the tax base is in region A, so it is better for A to increase tax rate.

- In equilibrium, t_A and t_B are too low, compared to the rates that maximize joint revenue. Essentially a coordination failure like we see in Cournot competitions.
- Remedies for such a failure
  - Centralization
  - Grants and transfers

World of Multiple Governments (cont'd)

- Avenue of research (horizontal tax competition)
  - Empirical:
    - Tax mimicking: if the theory is right (strategic complementarities between regions), then does the empirics back this result.
    - Compare centralized countries v.s. decentralized countries
    - Tax competition v.s. yard stick competition
      - Yard stick competition is the idea that voters will compare the performance of politicians in their own region to that of those in neighboring regions, and decide whether to re-elect the incumbent. This gives incentive for the incumbent to homogenize their policy with neighboring region.
  - Solution to tax competition
    - Centralization
    - Harmonization
    - Transfer and grant → with intention to solve the problem of tax competition
      - Equalization payment → with intention to redistribute resources across regions, or redistribute ability to collect taxes
  - Asymmetry and tax haven
    - Production function is $F(K, L_i)$. If L_i is small, then the capital-to-labor ratio is sensitive to the capital level, so that the tax base is very elastic. Such regions are more likely to reduce tax.
  - Heterogeneous capital, mobile v.s. immobile

- Vertical Tax Competition
  - The federal government sets tax rate t_F
  - The provincial government sets tax rate t_P
  - The tax base is common to both governments, $B(t_F, t_P)$ with $B_{t_F} < 0$ and $B_{t_P} < 0$
  - The (negative) externality is that the more one government taxes, the smaller the tax base is for the other government, and the governments don't take that into account. Thus, in this case the tax rates are going to be set too high.

  - Each government's problem

$$\max_{t_i} t_i B(t_i, t_j) \overset{\text{FOC}}{\Longrightarrow} \underbrace{B(t_i, t_j)}_{\substack{\text{direct benefit} \\ \text{from increasing tax}}} + \underbrace{t_i B_{t_i}(t_i, t_j)}_{\substack{\text{cost of} \\ \text{increasing tax}}} = 0 \;\Rightarrow\; -t_i \frac{B_{t_i}(t_i, t_j)}{B(t_i, t_j)} = 1$$

  - The first best:

$$\max_{t_i, t_j} t_i B(t_i, t_j) + t_j B(t_i, t_j) \overset{\text{FOC}}{\Longrightarrow} B(t_i, t_j) + (t_i + t_j) B_{t_i}(t_i, t_j) = 0$$

    From this, we see that at the decentralized level, the government doesn't take into account the negative externality generated by its taxing decisions, i.e. $t_j B_{t_i}(t_i, t_j)$.

  - Solutions
    - Centralization

- Transfer and grant
- Allocation of tax bases

Centralization v.s. Decentralization

- Do not confuse centralization with totalitarianism, and decentralization with democracy

- Cost of decentralization for (t or G)
  - Horizontal tax competition (with mobile tax base) → taxes will be too low, so will the provision of public goods
  - *Vertical tax competition*. This happens when there is a combination of both centralization and decentralization. There is no issue if there is <u>only</u> centralization or decentralization.
  - Spill-over benefits of public goods
    - Suppose there are two regions, A and B, that uses lump-sum taxes

$$A: \quad x + v(G_A) + \alpha_A v(G_B)$$
$$B: \quad x + v(G_B) + \alpha_B v(G_A)$$

In region A,

$$\max_{T,G_A} N_A x + N_A v(G_A) + N_A \alpha_A v(G_B), \qquad s.t. \begin{cases} w = x + T \\ cG_A = N_A T \end{cases}$$

which is equivalent to

$$\max_G N_A \left[w - \frac{cG_A}{N_A} \right] + N_A v(G_A) + N_A \alpha_A v(G_B) \overset{\text{FOC}}{\Longrightarrow} N_A v'(G_A) = c$$

Similarly, for region B, we have

$$N_B v'(G_B) = c$$

Thus public goods are efficiently provided <u>within each region</u>. The efficient level for the two regions as a whole, however, is given by

$$\max_{T_A,T_B,G_A,G_B} N_A x + N_A v(G_A) + N_A \alpha_A v(G_B) + N_B x + N_B v(G_B) + N_B \alpha_B v(G_A)$$

subject to

$$w_i = x_i + T_i, \qquad cG_i = N_i T_i, \qquad i \in \{A, B\}$$

The FOC is

$$N_i v'(G_i) + \alpha_{-i} N_{-i} v'(G_i) = c, \qquad i \in \{A, B\}$$

  - Return to scale. There are fixed costs to production of public goods, and decentralization implies that regions incur the fixed costs multiple times; whereas centralization can avoid this problem.
  - With centralization, for any level of public goods provision, we also have smaller tax burden per capita, because more people are paying taxes.

- Pork barrel politics
  - There is an election, with multiple riding (i.e. multiple representatives from localized regions), and first past the poll.
  - Suppose the country is divided into 5 regions, which can be aligned on a left-right scale.

L —|— 1 —|— 2 —|— 3 —|— 4 —|— 5 —|— R

  - Voter preference:
    - Intrinsic ideological preferences: $u_i \in [-0.5, 0.5]$, and voters are distributed uniformly with median c
    - Benefit from local spending $B(s_i)$

- Benefit of decentralization
  - Tibout argument: "vote with your feet"
    - Heterogeneous preferences $x + \alpha_i v(G_i)$, $i = A, B$, where $\alpha_A > \alpha_B$
    - A centralized government will provide G_i according to
$$N_A \alpha_A v'(G_A) + N_B \alpha_B v'(G_B) = c$$
      whereas each type of individual would prefer G_i to be provided according to
$$N_i \alpha_i v'(G_i) = c, \qquad i = A, B$$
  - Oates' decentralization theorem:
    - For a public good–the consumption of which is defined over geographical subsets of the total population, and for which the costs of providing each level of output of the good in each jurisdiction are the same for the central or for the respective local government–it will always be more efficient (or at least as efficient) for local governments to provide the Pareto-efficient levels of output for their respective jurisdictions than for the central government to provide *any* and uniform level of output across all jurisdictions (Oates, 1972, p. 35)
  - Information and moral hazard problem
    - Decentralized governments are going to have better information about the preferences of local citizens. So decentralized policy making is going to be better on average (if the central government makes a mistake in its policy, it affects all regions, but the mistake from a local government has less negative impact on the overall economy).
    - Yard stick competition: comparing performance of government officials across neighboring regions.

Review Class

- Practice Questions #4
  - Part A, Q2. In general, the statement is true. But if self-selection constraint is not binding, then we would have non-distorted marginal tax rate for either types.
  - Part B, Q2. Note that (a) is a Ramsey tax problem. At the end, all taxes are the same because the elasticities with respect to different taxes are the same.
  - Part B, Q3. Here (b) requires adding the self-selection constraint to the FOC's. If the self-selection constraint is not binding, then the solution is the same as before. If the constraint binds, then the first best is not attainable.

- Practice Questions #5
  - A1. Two ways to write the Samuelson rule:

$$Nv'(G) = c \iff v'(G) = \frac{c}{N}$$

    Note that this is the symmetric version of the Lindahl equilibrium.